

# *Production of Commodities by means of Commodities and Non-Uniform Rates of Profits* \*

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## **Abstract**

Sraffa's book *Production of Commodities by Means of Commodities* (PCMC) begins with the following:

*Let us consider an extremely simple society which produces just enough to maintain itself. Commodities are produced by separate industries and are exchanged for one another at a market held after the harvest (Sraffa, 1960a, p.3).*

One of Sraffa's objectives is to determine the

*... set of exchange-values which if adopted by the market restores the original distribution of the products and makes it possible for the process to be repeated; such values spring directly from the methods of production (Sraffa, 1960a, p.3).*

so that the system is in a *self-replacing state*. In this paper we study the *self-replacing* properties without assuming *uniform rate of profits*

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## Introduction

Sraffa's book *Production of Commodities by Means of Commodities* (PCMC) begins with the following:

*Let us consider an extremely simple society which produces just enough to maintain itself. Commodities are produced by separate industries and are exchanged for one another at a market held after the harvest (Sraffa, 1960a, p.3).*

The subtitle of the book indicates that Sraffa's contribution is a *A Prelude to a Critique of Economic Theory*. Sraffa's one of main aims seems to have been the tenability of the (*neoclassical*) postulate that relative commodity prices are a monotonic function of the profit rate (or of the wage rate). According to *neoclassical* requirements, as the profit rate increases and the wage rate decreases the price of a commodity of an industry having a high proportion of the value of the means of production to labour should, *ceteris paribus*, increase relative to the price of commodities where the proportion is low. This is well explained in Sraffa (1960a, Ch. 3, p. 14-5).

His investigation led to conclude that the notion of capital intensive techniques as opposed to labour intensive ones would be most of the times meaningless.

*... as the wages fall the price of the product of a low proportion ... may rise or it may fall, or it may even alternate in rising and falling, relative to its means of production; while the price of the product of a high-proportion ... industry may fall, or it may rise or it may alternate. What neither of such products can do ... is to remain stable in price relative to its means of production throughout any range, long or short, of the wage-variation.*

These results were at the core of the 60s' Two Cambridges famous Capital Controversy. If the prices of the individual products vary as the wage-rate varies, also the sum across all means of production necessary for the production of all the products will vary as the wage-rate varies. A conclusion of that debate was that the neoclassical aggregate production function may not always be well behaved because the value of aggregate capital may not be a monotonic function of the wage rate or of the profit rate.

The problem was acknowledged from a logical point of view by some neoclassical authors (Samuelson (1966) and Ferguson (1969)), who anyway considered it to be an empirically irrelevant problem (Sato, 1974). To the point that today there is very little debate on the empirical relevance of the aggregate neoclassical function as if aggregation was unproblematic. Recently Zambelli (2016), following a Sraffian methodology, has retraced the

60s' debate and has computed the neoclassical aggregate production function using empirical observations. The conclusion is that the aggregate production, as defined by Samuelson (1962), Arrow et al. (1961), does not have any empirical foundation, hence it does not exist and the logical problems exposed during the Two Cambridge Controversy cannot be confined to be just a curiosum because they seem to be the rule.

If Zambelli (2016) results hold and pass the scrutiny of academia, it follows that Sraffa's own work is not only important theoretically, but is of great importance also for empirical investigations.

An explicit assumption present in Sraffa's PCMC is that of a uniform rate of profits for all industries.

For the purposes of the *Prelude to a Critique of Economic Theory* the assumption of a uniform rate of profits may be sufficient. To go through the complications of non-uniform rate of profits is not necessary for the critique. In fact the Austrian School would maintain that the inverse of the profit rate would be correlated to the average period of production and this could be related with the *roundaboutness* of capital determining a relation with capital intensive methods<sup>1</sup>. Furthermore, the complications of non-uniform rate of profits are not necessary for the foundations of the critique to the incorporated labour theory of value embedded in Ch. 6 (*Reduction to Dated Quantities of Labour*) of PCMC. The reason being that the critique holds also for the general case. In other words, if the critique holds for the case of *uniform rate of profits* it would hold even more for *non-uniform rate of profits*.

It is Sraffa himself that considered the uniform rate of profits to be an assumption. In the text, at a very early stage, he writes that the profit rates “*must be uniform (Sraffa, 1960a, p.6, emphasis added)*”, but in the index he writes under the entry relative to the Rate of profits “*assumed to be uniform for all industries Sraffa (1960a, p.98, emphasis added)*”. Hence it is Sraffa himself that considered it to be an assumption<sup>2</sup>. To say that the

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<sup>1</sup>On this point Sraffa is crystal clear:

(The reduction to dated labour terms has some bearing on the attempts that have been made to find in the ‘period of ‘production’ an independent measure of the quantity of capital which could be used, without arguing in a circle, for the determination of prices and of the shares in distribution. But the case just considered seems to be conclusive in showing the impossibility of aggregating the ‘periods’ belonging to the several quantities of labour into a single magnitude which could be regarded as representing the the quantity of capital. The reversal in the direction of the movement of relative prices, in the face of unchanged methods of production, cannot be reconciled with any notion of capital as a measurable quantity independent of distribution and prices) (Sraffa, 1960a, p.38)

<sup>2</sup>Also the Italian edition of *Production of Commodities by Means of Commodities* refers to, in the Indice Analitico: *Saggio del profitto, supposto uniforme in tutte le industrie*

rate of profit is assumed to be uniform is very different than saying that it must. Hence it is not very clear what Sraffa really meant.

Much research has taken place having the *uniform rate of profits* as an assumption. Several authors claim that this is not really an assumption, but a property of the system a) determined by the methods of production and distribution (Sinha, 2010, 2011), or b) describing the long term convergent properties for the case of free competition (Kurz and Salvadori, 1995), or c) determining the *natural prices* around which the market prices gravitate (Gargnani, 1976; Garegnani, 1997).

We will remove the uniform rate assumption and study for the case of single-product industries and circulating capital the conditions in which the system could be in a *self-replacing* state.

We believe that in doing so we follow Sraffa's declared methods, which is the study of prices without changing the quantities. In the sequel we will develop an argument showing, we hope, that the assumption of the *uniform rate of profits* may be useful in certain contexts, but it is not a necessary one.

Another important implicit assumption is the absence, in Sraffa's market after the *harvest*, of any mechanism that might allow for trade to take place thanks to the use of deferred means of payments (i.e., endogenous debt and credit relations). In the presence of these means of payments it is still possible that the "[production] process be repeated", i.e. that the system repeats itself even when the prices are not *production prices*. We study the relation there is between the generation of these means of payments and the existing *methods of production*.

## 1 The self-replacing prices

*The significance of the equations is simply this: that if a man fell from the moon on the earth, and noted the amount of things consumed in each factory and the amount produced by each factory during a year, he would deduce at which values the commodities must be sold, if the rate of interest must be uniform and the process of production repeated. In short, the equations show that the conditions of exchange are entirely determined by the conditions of production. Sraffa(1927 or 1928, D3/12/7, emphasis added)<sup>3</sup>*

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(Sraffa, 1960b, p.127)

<sup>3</sup>Catalogue of Sraffa Papers, Wren Library, Trinity College, Cambridge, edited by Jonathan Smith. This quotation is from the archives of the Wren Library. It is dated between 1927 and 1928. See Gilibert (2006, p.28) and Gilibert (2003)

In the sequel we will compute these prices<sup>4</sup> for the general case in which the rates of interest (interpreted here as being the same as the rate of profits) may not be uniform<sup>5</sup>. Sraffa thought about the equations that came to be core of his 1960's book probably since his early works (Sraffa, 1925, 1926). From the Sraffa archives we see that he had written already in the twenties several equations trying to link the quantities used in production and the distribution of the physical surplus generated by the economic system with the theory of value. On the relation between the equations written and shown to Keynes in 1928 and PCMC see Gilibert (2003) and Gilibert (2006).

The quotation above is illuminating, especially when put in relation with the opening sentences of the opening section of PCMC. Sraffa principal scope in PCMC is to determine the

... set of exchange-values which if adopted by the market restores the original distribution of the products and makes it possible for the process to be repeated; such values spring directly from the methods of production (Sraffa, 1960a, p.3).

so that the system is in a *self-replacing state*.

The focus is on the determination of the prices and wages that, “*if adopted*”, will “*restore the original distribution of the product*”.

At the end of the harvest and before the market day the  $n$  producers have produced  $n$  commodities. In this market producers will sell their produced goods and the workers their labour.

## 1.1 Physical Quantities, Surplus and Distribution

### 1.1.1 Physical Quantities to be Exchanged

Without going into the details of the exchanges what is that we can say about the possibility of reproducing production and consumption as it did occur the previous year? We have some *objective* observations. Our privileged observer, *the man from the moon*, at the beginning of the market

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<sup>4</sup>Here it is assumed that in the market during the day the prices would be uniform. This assumption is is logically not strictly necessary. We might imagine that the exchanges during market operations, for which we and the *man from the moon* know nothing, take place bilaterally. In this case there can be exchanges of the same goods, let us say good  $i$  in exchange with good  $j$ , taking place at different prices. But we have to keep in mind that in our thought or mental experiment we consider hypothetical prices. In all the cases in which we have conceivable (bilateral) exchange prices there might be also be the case in which all the exchanges may take place at the same prices. These prices are obviously uniform.

<sup>5</sup>The whole of PCMC is a *mental* or *thought* experiment where the *constructive* method or mathematic is applied. In the sequel, given quantity observations – the same that the *man from the moon* can observe – we will explain how to compute the prices with an effective (i.e. algorithmic) way. On Sraffa's constructive method and *thought experiment* see Velupillai (1989, 2008).

day observes the produced  $b$ -quantities *brought* to the market. In the sequel we will focus on the prices that will restore the *original distribution of the product*.

$$[b_1, b_2, \dots, b_i, \dots, b_n]^T \quad (1.1)$$

$i = 1, 2, \dots, n$ . He also knows that there are, at the beginning of the market day, workers that are willing to work. Sraffa, in his thought experiment, gives to the *man from the moon* also information of the quantities that were used during the previous production period. He calls this information *the methods of production*. If the system has to reproduce itself the exchanges during the market day would have to be such that at the end of the market day the producers would have the means of production necessary to replicate the production process of the previous year.

$$b_i \xrightarrow{\text{exchange}} a_i^1, a_i^2, \dots, a_i^j, \dots, a_i^{n-1}, a_i^n, \ell_i \xrightarrow{\text{production}} b_i \quad (1.2)$$

where  $a_i^j$  is the mean of production  $j$  used in the production of good  $i$ , the quantity  $b_i$ . For the whole system this circularity is summarized with the following standard notation

$$\begin{array}{ccccccccccc} b_1 & \xrightarrow{\text{exchange}} & a_1^1 & \dots & a_1^j & \dots & a_1^n & \ell_1 & \xrightarrow{\text{production}} & b_1 \\ b_2 & \xrightarrow{\text{exchange}} & a_2^1 & \dots & a_2^j & \dots & a_2^n & \ell_2 & \xrightarrow{\text{production}} & b_2 \\ \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \xrightarrow{\text{production}} & \vdots \\ b_i & \xrightarrow{\text{exchange}} & a_i^1 & \dots & a_i^j & \dots & a_i^n & \ell_i & \xrightarrow{\text{production}} & b_i \\ \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \xrightarrow{\text{production}} & \vdots \\ b_n & \xrightarrow{\text{exchange}} & a_n^1 & \dots & a_n^j & \dots & a_n^n & \ell_n & \xrightarrow{\text{production}} & b_n \end{array} \quad (1.3)$$

These quantities may be written in compact matrix notation in the following way:

$$\mathbf{b} \xrightarrow{\text{exchange}} \mathbf{A}, \mathbf{L} \xrightarrow{\text{production}} \mathbf{b} \quad (1.4)$$

where:  $\mathbf{A}$  is an  $n \times n$  matrix whose components are the used means of production  $\{a_i^j\}$ ;  $\mathbf{L}$  is a  $n \times 1$  vector whose elements  $\{\ell_i\}$  is the labour used in production;  $\mathbf{b}$  is  $n \times 1$  vector whose element  $\{b_i\}$  is the harvest of good  $i$ <sup>6</sup>.

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<sup>6</sup>In the sequel we will indicate with bold lowercase letters the vectors, with the exception of the labour vector  $\mathbf{L}$  and the physical surplus vector  $\mathbf{S}$ , and uppercase letter matrices. To simplify notation the the row  $i$  of a matrix, let us take as example matrix  $\mathbf{A}$ , would be written in bold in the following way,  $\mathbf{a}_i$ , while the column  $j$  will be written as  $\mathbf{a}^j$ . Furthermore we will assume all the values to be rational numbers. This is a question of realism (quantities and prices are observed always to be rational numbers with finite number of digits), but in this context to work with rational numbers means to be in principle as accurate as possible. Sum, subtraction, multiplication, division between

In this article we will focus our attention to the case in which all commodities are *basic* commodities as defined by Sraffa<sup>7</sup>.

### 1.1.2 Surplus available for distribution or consumption

Note that once we have the observed and hence known quantities of eq. 1.3 or eq. 1.4 we know also the Physical Surplus that was produced during the production period. That is

$$\begin{aligned}
 s_1 &= b_1 - \sum_{i=1}^n a_i^1 \\
 s_2 &= b_2 - \sum_{i=1}^n a_i^2 \\
 &\vdots \\
 s_j &= b_j - \sum_{i=1}^n a_i^j \\
 &\vdots \\
 s_n &= b_n - \sum_{i=1}^n a_i^n
 \end{aligned} \tag{1.5}$$

where  $s_i$  is the surplus of commodity  $i$  available for distribution after the quantities  $\{a_i^j\}$  have been put aside for the next year production or, alternatively, is the quantity produced in the previous period which is left once the inputs used in production have been removed. In compact matrix notation we have:

$$\mathbf{S} = (\mathbf{B} - \mathbf{A})^T \mathbf{e} \tag{1.6}$$

where:  $\mathbf{e}$  is an  $n \times 1$  unit or summation vector (each element is 1);  $T$  is the transpose operator;  $\mathbf{S}$  is the  $n \times 1$  Physical Surplus vector or Physical *Net National Product*;  $\mathbf{B}$  is an  $n \times n$  diagonal matrix having as elements in the diagonal the elements of gross production  $\mathbf{b}$  and the other elements are 0s.

### 1.1.3 The *Man from the Moon* and the computation of *Self-Replacing Prices*

The *man from the moon* has, before the opening of the market, to compute the prices that will allow the system to replicate for next year (or this year) what happened the current year (or last year). After the harvest and before the market day begins we have only the produced quantities  $\mathbf{b}$ . These quantities could be transformed through trade, at the end of the market day, into a different allocation of resources.

That is, we have that exchange may reverse the process. The whole circular process could become

$$\mathbf{A}, \mathbf{L} \xrightarrow{\text{production}} \mathbf{b} \xrightarrow{\text{exchange}} \mathbf{A}, \mathbf{L}, \mathbf{S} \tag{1.7}$$

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rational numbers could be made with absolute precision. Also the inverse of a matrix whose elements are rational numbers can be made with absolute precision (Aberth, 2007, Ch.9).

<sup>7</sup>a

provided there are uniform prices that restore the quantities and hence the distribution of means of production and of the surplus that were produced at the beginning of the production or transformation period.

## 1.2 Definition of the self-replacing condition.

Our *man from the moon* has the task of finding the vector of prices and wage rate that would allow self-reproduction of the system as in 1.7 . In synthesis the prices would have to be such that the exchange process during the market day would lead to the following distribution of the gross product available (and employment):  $\mathbf{b} \xrightarrow{\text{exchange}} \mathbf{A}, \mathbf{L}, \mathbf{S}$ . The quantity  $\mathbf{S}$  is consumed while the means of production  $\mathbf{A}$  and the  $\mathbf{L}$  could be used to produce the output ( $\mathbf{A}, \mathbf{L} \xrightarrow{\text{production}} \mathbf{b}$ ).

Given a vector of prices and a wage rate we would have the following:

$$\begin{array}{rcl}
 a_1^1 p_1 + \dots + a_1^j p_j + \dots + a_1^n p_n + \ell_1 w & \leq & b_1 p_1 \\
 a_2^1 p_1 + \dots + a_2^j p_j + \dots + a_2^n p_n + \ell_2 w & \geq & b_2 p_2 \\
 \vdots + \vdots + \vdots + \vdots + \vdots + \vdots & \geq & \vdots \\
 a_i^1 p_1 + \dots + a_i^j p_j + \dots + a_i^n p_n + \ell_i w & \leq & b_i p_i \\
 \vdots + \vdots + \vdots + \vdots + \vdots + \vdots & \geq & \vdots \\
 a_n^1 p_1 + \dots + a_n^j p_j + \dots + a_n^n p_n + \ell_n w & \leq & b_n p_n
 \end{array} \tag{1.8}$$

which in matrix notation could be written as:

$$\mathbf{A}\mathbf{p} + \mathbf{L}w \begin{array}{l} \leq \\ \geq \end{array} \mathbf{B}\mathbf{p} \tag{1.9}$$

The left hand sides of eqs. 1.8 and 1.9 would be the individual industries expenditures while the right hand side the industries revenues.

Clearly if there are some sectors, let us sector  $j$  for which we have  $\mathbf{a}_j \mathbf{p} - \ell_j w > b_j p_j$  the system would not be able to reproduce simply because the industry  $j$  would not have the necessary purchasing power to buy all the necessary means of production. In the sequel we will consider only the case for which the prices are such that  $\mathbf{A}\mathbf{p} + \mathbf{L}w \leq \mathbf{B}\mathbf{p}$ <sup>8</sup>. This is our self-replacing condition.

We can write this explicitly. Associated to each industry  $i$  and the workers the revenues are<sup>9</sup>:

<sup>8</sup>But industry  $j$  may be able to buy means of production if the owners of these means of production are willing to exchange against a future promise to pay. In a companion paper this case will be discussed in depth.

<sup>9</sup>Here and in the sequel with *Revenues* we mean the values of actual sale of the physical quantities or labour and with *Expenditures* we mean the value of the physical quantities bought. The necessity of this clarification will become obvious in the sequel when credit and debt relations are introduced.

$$\begin{aligned} \text{Revenues}(i) &= b_i p_i \quad i = 1, \dots, n \\ \text{Revenues}(n+1) &= \mathbf{e}^T \mathbf{L} w \end{aligned} \quad (1.10)$$

While the expenditures are:

$$\begin{aligned} \text{Expenditures}(i) &= \mathbf{a}_i \mathbf{p} + \ell_i w + \mathbf{h}_i \mathbf{p} \quad i = 1, \dots, n \\ \text{Expenditures}(n+1) &= \mathbf{h}_{(n+1)} \mathbf{p} \end{aligned} \quad (1.11)$$

where  $\mathbf{h}_i$  is the consumption vector of producer  $i$  and  $\mathbf{h}_w$  is the consumption vector of the workers<sup>10</sup>. Clearly those industries that are in the condition for which the expenditures would be higher than the revenues would not have the purchasing power to buy the means of production necessary to replicate the production of the previous period. These industries are in a condition of potential financial deficit. They would be able to purchase the necessary means of production only by agreeing to a deferred payment to take place during the years to follow in favor of the industries in potential surplus. Recall that the current exercise is to study the conditions that would allow the economic system to be in a self-replacing state as described above and summarized in the relation 1.7. If the quantities  $\mathbf{A}, \mathbf{L}, \mathbf{b}$  have to be restored for given prices, we must have that in general<sup>11</sup> the inequalities of eq. 1.8 may be “*eliminated*” if revenues of each agent are equal to their expenditures.

**Definition of Self-Replacing Condition.** The system is in a self-replacing state when, see eqs. 1.10 and 1.11, the

$$\text{Revenues}(z) \equiv \text{Expenditures}(z) \quad \forall z = 1, \dots, n+1 \quad (1.12)$$

with  $z$  being the index of the  $n$  industries and the workers.

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<sup>10</sup>In the appendix we discuss the details of the distribution of the physical surplus  $S$ . In order to keep the argument as closed as possible to the standard literature on Sraffa’s schemes we avoid here to discuss in depth the issue of physical distribution of the surplus, see the Appendix at p.33.

<sup>11</sup>Here the qualification — “*in general*” is put to imply that the system could be in a self-replacing state if there were lending and borrowing possibilities. In that case there could be exchanges taking place at prices for which the revenues are less than the expenditures for some agents and consequently there would be other agents for which the revenues would be higher for others. In a self-replacing condition this could happen if and only if those who do not have the necessary purchasing power would borrow in the form of I Owe You. Alternatively the expenditures of those in lack of purchasing power would have to be reduced and consequently also the revenues of those in in potential excess of purchasing power would have to be lower. This being the case would imply that eq. 1.10 and 1.11 would not hold.

The accounting balances would require that for given prices we have:

$$\begin{aligned}
(1 + r_1)\mathbf{a}_1\mathbf{p} + \ell_1 w &= b_1 p_1 \\
(1 + r_2)\mathbf{a}_2\mathbf{p} + \ell_2 w &= b_2 p_2 \\
&\dots &= \dots \\
(1 + r_i)\mathbf{a}_i\mathbf{p} + \ell_i w &= b_i p_i \\
&\dots &= \dots \\
(1 + r_n)\mathbf{a}_n\mathbf{p} + \ell_n w &= b_n p_n
\end{aligned} \tag{1.13}$$

In matrix notation the above eqs. 1.13 become:

$$(\mathbf{I} + \mathbf{R})\mathbf{A}\mathbf{p} + \mathbf{L}w = \mathbf{B}\mathbf{p} \tag{1.14}$$

where:  $\mathbf{R}$  is a diagonal matrix whose diagonal elements are the single industries profit rates,  $r_1, r_2, \dots, r_i, \dots, r_n$  (vector  $\mathbf{r}$ )<sup>12</sup>.

### 1.3 The Physical surplus or net national product as *numéraire*

It is convenient to measure prices,  $\mathbf{p}$  in terms of the purchasing power of the Physical Surplus or Physical Net National Product. Once the *numéraire* is picked to be the Surplus  $\mathbf{S}$  we have by definition that the following relation should hold:

$$\mathbf{S}^T \mathbf{p} = \mathbf{e}^T (\mathbf{B} - \mathbf{A})\mathbf{p} = 1 \tag{1.15}$$

This simplifies the analysis without changing the substance of the argument. Relative price ratios do not change as the *numéraire* changes. Therefore one can shift from one *numéraire* to another without having to change the accounting relation or other things<sup>13</sup>. But most importantly with this particular choice for the *numéraire* we have that the wage rate  $w$ , under certain conditions, could also be interpreted as the share of the physical surplus that goes to workers. Furthermore, as it will be shown in the next section because the value of the surplus would be 1, we also have that the Share of the Surplus has the same numerical value as the measured quantities.

<sup>12</sup>The *accounting* implicit in eqs.1.13 1.14 is made consistent with the choice made in PCMC. We think that the alternative choice of computing the profit rates as including also labour costs would be simpler and more appropriate. Nevertheless the qualitative conclusions would not change. The difference is that, for example, eq. 1.13 would have to be written as  $(\mathbf{I} + \mathbf{R})(\mathbf{A}\mathbf{p} + \mathbf{L}w) = \mathbf{B}\mathbf{p}$ . The proposition made in the sequel of the article may be appropriately modified to consider this different accounting. We leave an analysis of the consequences of these alternative to another article.

<sup>13</sup>Another obvious choice for the *numéraire* could be the Standard Commodity. But while the standard commodity was important for the purpose of PCMC, here it is not essential. The reason being that we are searching for the vectors of prices that would allow the economic system to self-reproduce. For this purpose the choice of the *numéraire* does not really matter. Naturally it is also Sraffa himself that did give to the Net National Product or surplus great importance (Sraffa, 1960a, Sec.12, p.11).

#### 1.4 Shares of the surplus and the computation of prices and rates of profits.

We are now in the position of computing the prices that would allow the system to replicate. We have the “*objective*” observations of the quantities or “*things*”, produced and available at the beginning of the market day: the gross product  $\mathbf{b}$ . Given the knowledge of used past methods,  $\mathbf{A}$  and  $\mathbf{L}$ , and the physical surplus  $\mathbf{S}$  we can compute the prices that would allow replication for next year.

The distribution of the surplus to the different industries and to the workers would depend on the prices. For given prices and wage rate we have the following accounting identity:

$$\mathbf{dS}^T \mathbf{p} = \begin{array}{c} \text{Distribution of} \\ \text{Surplus or NNP} \\ \left[ \begin{array}{c} d_1 \mathbf{S}^T \mathbf{p} \\ d_2 \mathbf{S}^T \mathbf{p} \\ \vdots \\ d_n \mathbf{S}^T \mathbf{p} \\ d_w \mathbf{S}^T \mathbf{p} \end{array} \right] \end{array} = \begin{array}{c} \text{Purchasing Capacity} \\ \text{Income} \\ \left[ \begin{array}{c} b_1 p_1 - \mathbf{a}_1 \mathbf{p} - \ell_1 w \\ b_2 p_2 - \mathbf{a}_2 \mathbf{p} - \ell_2 w \\ \vdots \\ b_n p_n - \mathbf{a}_n \mathbf{p} - \ell_n w \\ \mathbf{e}^T \mathbf{L} w \end{array} \right] \end{array} \quad (1.16)$$

where  $d_1 \mathbf{S}^T \mathbf{p}$ ,  $d_2 \mathbf{S}^T \mathbf{p}$ ,  $\dots$ ,  $d_n \mathbf{S}^T \mathbf{p}$  are the shares of the Newt National Product going to the  $n$  industries and  $d_w \mathbf{S}^T \mathbf{p}$ , is the share going to labour.

Recalling that  $\mathbf{S}^T \mathbf{p} = 1$ , see eq.1.15 the above accounting identity eq. 1.16, may be rewritten as:

$$\mathbf{d} = \begin{array}{c} \text{Share of} \\ \text{Surplus or NNP} \\ \left[ \begin{array}{c} d_1 \\ d_2 \\ \vdots \\ d_n \\ d_w \end{array} \right] \end{array} = \begin{array}{c} \text{Purchasing Capacity} \\ \text{Income} \\ \left[ \begin{array}{c} b_1 p_1 - \mathbf{a}_1 \mathbf{p} - \ell_1 w \\ b_2 p_2 - \mathbf{a}_2 \mathbf{p} - \ell_2 w \\ \vdots \\ b_n p_n - \mathbf{a}_n \mathbf{p} - \ell_n w \\ \mathbf{e}^T \mathbf{L} w \end{array} \right] \end{array} \quad (1.17)$$

Written in a more compact notational form and recalling that  $\mathbf{S}^T \mathbf{p} = 1$ , see eq.1.15 the above accounting identity eq. 1.17, may be rewritten as:

$$\mathbf{d} = \left[ \begin{array}{c|c} (\mathbf{B} - \mathbf{A}) & -\mathbf{L} \\ \hline \mathbf{0}_{1 \times n} & \mathbf{e}^T \mathbf{L} \end{array} \right] \left[ \begin{array}{c} \mathbf{p} \\ w \end{array} \right] \quad (1.18)$$

The distributional vector  $\mathbf{d}$  is the distribution of the produced surplus  $\mathbf{S}$  among producers and workers in value terms but it can also be interpreted as an equivalent of a fraction of the physical surplus  $\mathbf{S}$ <sup>14</sup>.

<sup>14</sup>In the Appendix an explanation of how to go, in this context, from shares in value to physical shares equivalents is provided.

As we can see from equations 1.16, 1.17 and 1.18, there is a unique relation between the price vector  $\begin{bmatrix} \mathbf{p} \\ w \end{bmatrix}$  the distribution  $\mathbf{d}$ . This is, in our view, in line with what Sraffa meant at the beginning of PCMC when he wrote that “*There is a unique set of exchange-values which if adopted by the market restores the original distribution of the products ...*”(Sraffa, 1960a, p.3, emphasis added). Which is both distribution in value and in quantities<sup>15</sup>.

Equations 1.17 and 1.18 are our fundamental equations. Once a set of prices and a wage rate are given we can check whether these prices and wage rate are sufficient for the system to be in a self-replacing state. This self-replacing condition is verified when the elements of  $\mathbf{d}$  are equal or greater than 0 and their sum is equal to 1<sup>16</sup>.

This is an important check, but it is not in itself a constructive way to determine the prices. The *man from the moon* could just proceed with trial and error procedure and never figure out the prices.

A constructive solution of the problem would be given by the set of prices and wage rate associated with a arbitrarily picked distribution vector  $\mathbf{d}$ <sup>17</sup>.

$$\begin{bmatrix} \mathbf{p} \\ w \end{bmatrix} = \begin{bmatrix} (\mathbf{B} - \mathbf{A}) & -\mathbf{L} \\ \mathbf{0}_{1 \times n} & \mathbf{e}^T \mathbf{L} \end{bmatrix}^{-1} \mathbf{d} \quad (1.19)$$

Once the inverted matrix is expanded we have the following:

$$\begin{bmatrix} \mathbf{p}_d \\ w_d \end{bmatrix} = \begin{bmatrix} (\mathbf{B} - \mathbf{A})^{-1} & (\mathbf{B} - \mathbf{A})^{-1} \mathbf{L} \\ \mathbf{0}_{1 \times n} & \frac{1}{\mathbf{e}^T \mathbf{L}} \end{bmatrix} \mathbf{d} \quad (1.20)$$

The major difference between this equation (eq. 1.19) and 1.18 is that once  $\mathbf{d}$  is picked we know for sure that the computed prices and wage would be such as to allow self-replacing. We could not say the same, without further investigations on the properties of the system, if we picket prices and wage rate arbitrarily<sup>18</sup>. Once we have determined the prices coherent with a

<sup>15</sup>The case in which there is no surplus to be distributed – as it would be that of Chapter 1 of PCMC, *Production for Subsistence* – we would have a special case in which all the entries of  $\mathbf{d}$  are 0s. This would be a condition where there is a degree of freedom for the determination of prices [It would be a null-space]. But once we observe the existence of a Surplus uniqueness is established.

<sup>16</sup>If the above conditions for the vector  $\mathbf{d}$  are verified this also implies that  $\mathbf{S}^T \mathbf{p} = 1$

<sup>17</sup>The problem is that once a triple  $\mathbf{b}, \mathbf{A}, \mathbf{L}$  is given we would have to find out the domain for the prices and the wage rate so as to allow self-replacing conditions. The share vector  $\mathbf{d}$  has elements that are bounded from below (value 0) and from above (value 1). With the sum of all elements being 1

<sup>18</sup>Which can be written explicitly in terms of the distribution and financial balances of the industries and of the workers:

$$\mathbf{p}_d = (\mathbf{B} - \mathbf{A})^{-1} \mathbf{d}_{n \times 1} + (\mathbf{B} - \mathbf{A})^{-1} \frac{\mathbf{L}}{\mathbf{e}^T \mathbf{L}} d_w \quad (1.21)$$

given distribution we can determine the profit rates associated with that distribution.

$$\mathbf{R}\mathbf{A}\mathbf{p}_d = (\mathbf{B} - \mathbf{A})\mathbf{p}_d - \mathbf{L}w_d \quad (1.23)$$

By performing element by element division  $\oslash$  of the right hand side and left hand side we obtain the vector of profit rates  $\mathbf{r}$ :

$$\mathbf{r}_d = \left( (\mathbf{B} - \mathbf{A})\mathbf{p}_d - \mathbf{L}w_d \right) \oslash (\mathbf{A}\mathbf{p}_d) \quad (1.24)$$

$$\mathbf{R}_d = \text{diag}(\mathbf{r}_d) \quad (1.25)$$

### Determination of all the prices, wage rates and profit rates that allow for the Self-Replacing condition.

We have a constructive way to determine the set (or space) of prices, wage rate and profit rates that allow the self-replacing condition as defined above at page 9. We can span the solution space by computing the values associated with all possible (discrete) combinations of the distribution vector  $\mathbf{d}$  and in this way determine the set of possible solutions<sup>19</sup>.

## 1.5 Distribution, prices and wage rate once the profit rates are given.

In the previous section we have explained how to generate the space or set of all possible self-replacing solutions. If we were given knowledge about an arbitrarily picked vector of profit rates we could check whether it belong to the set of solutions by searching an equal vector from the space of solutions.

An alternatively, once the vector of profit rates,  $\mathbf{r}$  is given we know that there could be a vector of prices  $\mathbf{p}$  and a wage rate  $w$  that could be associated to it by putting relations 1.14 and 1.15 together:

$$\left[ \begin{array}{c|c} \mathbf{B} - (\mathbf{I} + \mathbf{R})\mathbf{A} & -\mathbf{L} \\ \hline \mathbf{e}^T(\mathbf{B} - \mathbf{A}) & 0 \end{array} \right] \left[ \begin{array}{c} \mathbf{p} \\ w \end{array} \right] = \left[ \begin{array}{c} \mathbf{0}_{n \times 1} \\ 1 \end{array} \right] \quad (1.26)$$

where:  $\mathbf{p}_d$  is the vector of prices consistent or determined by the distribution  $\mathbf{d}$  of the surplus  $\mathbf{S}$ ;  $\mathbf{d}_{n \times 1}$  is the distribution among the  $n$  industries;  $d_w$  is the distribution of the surplus to the workers – given our choice of

$$w_d = \frac{1}{\mathbf{e}^T \mathbf{L}} d_w \quad (1.22)$$

<sup>19</sup>Given that our domain is in terms of (finite digits) rational numbers what we mean here with the phrase *all possible (discrete) combinations of the distribution vector  $\mathbf{d}$*  is in terms of a desired precision factor. Given any desired level of precision we would have a finite number of vectors  $\mathbf{d}$ . This set of vectors is an enumerable set. All possible values can be computed associated to each of the vectors belonging to the set. For example, if the desired or observed precision is given by the number  $m$ , the number of distributional vectors is an integer which is a function of  $m$  and  $n + 1$ . For example in the case of three commodities and one digit precision the number of distributional vectors is 275, with two digits precision is 173340 and with three digits precision 167605051 and so on.

As we discussed in the previous section for the case of arbitrarily picked prices, it is not the case that arbitrarily picked profit rates would be associated with self-replacing profit rates.

A price vector necessary to allow the system to reproduce the previous year production would be

$$\begin{bmatrix} \mathbf{p}_r \\ w_r \end{bmatrix} = \begin{bmatrix} \mathbf{B} - (\mathbf{I} + \mathbf{R})\mathbf{A} & -\mathbf{L} \\ \mathbf{e}^T(\mathbf{B} - \mathbf{A}) & 0 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0}_{n \times 1} \\ 1 \end{bmatrix} \quad (1.27)$$

if the shares of the surplus are such that

$$\mathbf{d}_r = \begin{bmatrix} (\mathbf{B} - \mathbf{A})\mathbf{p}_r - \mathbf{L}w_r \\ \mathbf{e}^T \mathbf{L}w_r \end{bmatrix} \quad (1.28)$$

and ( $d_z \geq 0 \quad z \in [1, n + 1]$ ) and ( $\sum_{z=1}^{n+1} d_z = 1$ )<sup>20</sup>.

## 2 Self-Replacing without uniform rate of profits. Some examples.

What we have discussed above is inside the conventional and traditional literature associated with Sraffian Schemes. The only difference being, with respect to what normally found in the literature, the absence of the imposition of a uniform rate of profits. The uniform rate of profits is here just a special case.

Above we have seen that when the distribution of the shares of the surplus is given we have that the prices, wage rate and profit rates are determined (eqs. 1.21, 1.22 1.24). And the same is also true, provided proper boundaries, for the opposite: when the profit rates are given, prices and wage rates are given and a unique distribution of the shares of the surplus is given.

Let us take an example using the numbers in PCMC, page 19.

$$\mathbf{A} = \begin{bmatrix} 90 & 120 & 60 \\ 50 & 125 & 150 \\ 40 & 40 & 200 \end{bmatrix} \quad \mathbf{L} = \begin{bmatrix} \frac{3}{16} \\ \frac{5}{16} \\ \frac{8}{16} \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 180 \\ 450 \\ 480 \end{bmatrix} \quad (2.1)$$

<sup>20</sup>Clearly  $\mathbf{d}_r$  is not necessary equal in dimension to  $\mathbf{d}$ .  $\mathbf{d}_r$  has as dimension  $n + 1$  (number of commodities or industries plus workers) while the vector of individual shares  $\mathbf{d}$  can have any dimension going from the lowest (which would be the same dimension as  $\mathbf{d}_r$  and the highest that would be equal to the number of producers (which might be higher than the number of industries) plus the number of total workers (see below fn. 25). For the simplicity of the exposition we assume here that  $\mathbf{d}_r$  has the same dimension as  $\mathbf{d}$  and hence  $\mathbf{d}_r \equiv \mathbf{d}$ . In a way  $\mathbf{d}_r$  can be seen as a functional distribution of income while  $\mathbf{d}$  could be the population distribution of income. Example 1 below is a case where  $\mathbf{d}_r$  and  $\mathbf{d}$  coincide, while example 2 below is a case where there are two different distribution vectors,  $\mathbf{d}_r$  and  $\mathbf{d}$ . The reason being that the industries are three while the total population is made of 4 producers and 16 workers.

The first row would be the iron industry, the second the coal industry and the third the wheat industry. The columns of  $\mathbf{A}$  would indicate the means of production used as inputs by the industries. The first column would be iron, the second coal and the third wheat. These are the quantities we observe. At the end of the production period producers have produced quantities  $\mathbf{b} = [180, 450, 480]^T$  which have to be exchanged to organize production. We do not know what would happen during the market day, but if things have to be done like the previous year we know that at the end of the market day the surplus to be distributed would be  $\mathbf{S} = [0, 165, 70]^T$ .

Clearly the system would be in a self replacing state for any distribution vector  $\mathbf{d}$ . If we were given also the information about the distribution the prices would be uniquely determined. Also in the case in which a uniform rate of profits was given the prices would be uniquely determined.

### 2.1 Example: functional income distribution. Comparison of different set of prices when the Share of Surplus to workers is zero (Sraffa, 1960a, p.19)

At the beginning of the market day, after the harvest, the iron industry has 180 tons of iron, the coal industry has 450 tons of coal and the wheat industry 480 tons of wheat. This is the only things with exchange values. We could say that this is the only *capital* they have. These things have to be exchanged with the other commodities. If the share of output to workers is zero it means that the workers are “*slaves*” being paid zero wage. Therefore the producers would not have to worry about the cost of labour.

We, as observers and the *man from the moon* do not know what the trading prices would be. This would depend on what would happen in the market.

An analytical tool is to look for self-replacing prices. That is those prices will allow the producers to sell their products so as being able to produce the same output next year. This means that the revenues must be equal to the expenditures as described above with equation 1.12.

As we have seen above there is a huge number of price vectors, wage rates and distributions that would allow for the system to be in the self-replacing state. Once a vector of prices and a wage rate is found we have a unique set of profit rates and a unique distribution associated to it.

As an example let us consider two different set of prices a wage rate or share of the surplus to the workers to be 0 (i.e.  $d_w = 0$ ).

The first case is one in which the prices are associated with a uniform rate of profits. An inspection to table 2.1 shows that shares of the surplus – which is also the level of profits – are not uniform.

The second case is one in which the prices are associated with an even distribution of the shares. An inspection to table 2.1 shows that in this case

Table 2.1: *Share of Surplus to Workers = 0*

	Uniform Rate of Profits			Uniform Distribution in Shares		
	Prices	Prof.Rates	Shares	Prices	Prof.Rates	Shares
Iron	12.64*	20.0 %	37.9 %	12.15*	18.0 %	33.3 %
Coal	4.59*	20.0 %	34.5 %	4.54*	19.5 %	33.3 %
Wheat	3.45*	20.0 %	27.6 %	3.57*	24.1 %	33.3 %
Workers	0	0	0	0	0	0

(\*) Multiplied by  $\times 10^{-3}$

it is the rates of profits that are not uniform.

The argument which is often used to justify the assumption of the uniform rate of profits is that of free competition: *capitalists* will seek to get the “highest” profits and this would lead to bid prices up and down until the rate of profit is uniform.

Here we have a dilemma or seemingly a paradox. Please note that in the example of table 2.1 at the uniform rate of profit (20%) corresponds an uneven distribution among the industries of the surplus  $\mathbf{S}$  and at an even distribution among the industries of the surplus corresponds non uniform rates of profits.

One could argue that the *capitalists* would prefer to invest their capital in the sector which would give the highest profit level or income. This industry is the iron industry (37.93%). This should lead towards a fall of the price of iron relative to the prices of the other two commodities because *capitalists* may want to maximize profits, so the argument often goes. But this would determine a change in the distribution, that is a change in the profit levels. With the new prices there might be other sectors that would have higher shares. Hence the *capitalists* will seek to get these higher shares by investing in these sectors. But the prices of these sectors would get to be lower and that of the iron would get to be higher. A *free competition* argument could lead to conclude, we think, that the system would settle, given an unchanged share to the workers, to the case in which we have a uniform distribution of the shares among the industries. But as we can see from table 2.1 this solution is associated with a very different set of profit rates, that are far from being uniform (18.0 % for the iron industry, 19.5 % for the coal industry and 24.1 % for the wheat industry). The case in which we have a uniform rate of profits is what is defined in the literature as a classical free competition case (see on this Kurz and Salvadori (1995, Ch.1), Stigler (1987)).

But in Sraffa’s book there is no mention of long-period or free competition solutions.

Table 2.2: *Egalitarian per capita Distribution*

	Egalitarian Distribution in Shares			Uniform Rate of Profits		
	Shares	Prof.Rates*	Prices**	Shares	Prof.Rates	Prices
Iron	5.0 %	2.90 %	10.70*	7.27 %	4.16 %	10.94*
Coal	5.0 %	2.99 %	4.39*	6.98 %	4.16 %	4.44*
Wheat	10.0 %	7.18 %	3.94*	5.75 %	4.16 %	3.83*
Workers	80.0 %	–	0.80	80.0 %	–	0.80

The distribution in shares is here proportional to the population. This case is the one in which there is 1 producer in the iron industry, 1 in the coal industry, 2 in the wheat industry, and 16 workers.

(\*) Multiplied by  $\times 10^{-3}$

## 2.2 Example: personal income distribution. “Egalitarian” distribution according to population (Sraffa, 1960a, p.19).

In the previous section we have seen that it is not clear at all whether the free competition solution would be the one in which we have *uniform distribution id shares* or *uniform rates of profits*.

Let us see now a different example where we consider the total population spit into workers and producers. Let us assume that in the economic system of page 19 of PCMC we add the information that there are 16 workers, and 2 industrialists (one producing iron and the other producing coal) and two farmers producing wheat. What are the prices, profit rates and distribution to be preferred? In table 2.2 we compare an *egalitarian* (or *cooperative*) set of prices and profit rates with respect with the distribution that would be implied for the case in which we have a uniform rate of profits. What we mean here by egalitarian distribution of the physical surplus is the one in which the share is the same for the 16 workers and the 4 producers. Let us now compare this situation with that in which the rates of profits are uniform.

The producers as a whole have 20.0% of the physical surplus. The situation in which the rate of profits is uniform is not satisfactory for the coal industrialist and the two farmers. They would get a higher share if they were producing iron. As in the case of the previous section, we might expect the price of iron to go down and so on and so forth. As in the previous section. It is not clear at all what the free competition solution would be. But clearly in both examples we have that the uniform rate of profits solutions would be associated to different distribution in shares.

### 2.3 Distribution in shares, wage-profit curves, average of profit rates.

In the previous sections we have developed two different examples. Let us now consider the general case. As we have seen from eqs. 1.21, 1.22 1.24 once the distribution is given we have a unique set of prices, wage rate and profit rates.

Given any triple  $\mathbf{A}, \mathbf{L}, \mathbf{b}$  the prices that would allow the system to be in a self-replacing state are all the combinations for which each element of the distribution in shares vector is greater or equal zero ( $d_z \geq 0 \quad z \in [1, n+1]$ ) and the sum of all the elements is 1 ( $\sum_{z=1}^{n+1} d_z = 1$ ). It is the case that for any given distribution, there is a unique price vector,  $\mathbf{p}_d$ , wage rate  $w_d$  and vector of rates of profits  $\mathbf{r}_d$ . Figure 2.1 is the domain of the price vector associated to a given share of the surplus to the workers. There many price vectors that would be associated with the same distribution of the surplus to the workers. Clearly the only case in which there is only one price vector is when the share of the surplus to the workers is 1 (and hence nothing is left to the producers). Figure 2.2 is a different way of presenting the same information as in fig.2.1. The prices are here in terms of wheat (i.e.  $p_{wheat} = 1$  and not as in fig.2.1 where they are in terms of the surplus as *numéraire*). The bigger triangle reports the domain for the prices of iron and of coal in the case that the system has to be in a self-replacing state and the share to workers is zero. Couples of prices of iron and wheat belonging to the triangle would be self-replacing prices. Couples outside the domain would be nor self-replacing. As the share to workers increase the domain for the self-replacing prices decreases, as it is to be expected.

Still using the values of (Sraffa, 1960a, p.19). (see above eq. 2.1) Figure 2.3 reports the maximum rate of profits as a function of the share of the surplus to worker going from 0 to 1 ( $d_w \in [0, 1]$ ). The maximum rate of profit per industry  $i$  and for a given distribution to labour  $d_w$  would be realized when the distribution do the other industries is zero. In that case also the rate of profits of the these remaining industries would be zero. But the system has a whole would be in a self-replacing state.

As we can see from Figure 2.3 these maximum rate of profits do decrease, as it would have to be expected, as the share to workers increase. Given a wage rate there is a wide variety of combinations of profit rates.

The lowest curve is the traditional wage-profit curve associated with the uniform rate of profits. The set in which the rates of profits are uniform ( $r_1 = r_2 = \dots = r_i = r_n$ ) is a subset of this bigger set. Traditionally it is this subset that has been studied.

Figure 2.4 shows an important computation made in this paper. It is the distribution of the share to the industries associated with the uniform rate of profits case. As it is clear from the figure if the rate of profit is uniform. so

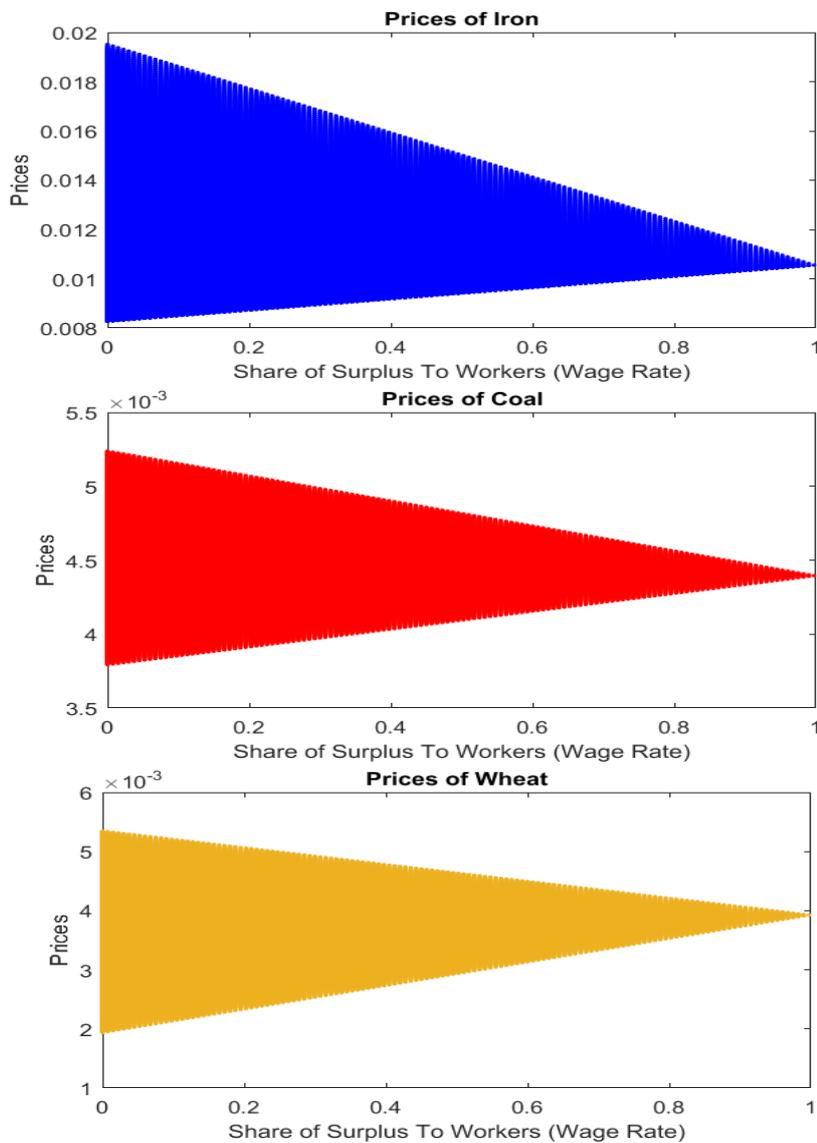


Fig. 2.1: *Example. Set of Prices that would allow the system to be in a self-replacing state.* The three figures above report the set of prices that would allow the system to be in a self-replacing state. Once the share of the surplus to the workers is given prices would have to be inside the triangle. Clearly the prices are linearly dependent, once the wage rate, and two of the prices are given also the third price would be given. The values of  $\mathbf{A}$ ,  $\mathbf{L}$ ,  $\mathbf{b}$  used for this figure are reported above in 2.1 (Sraffa, 1960a, p.19).

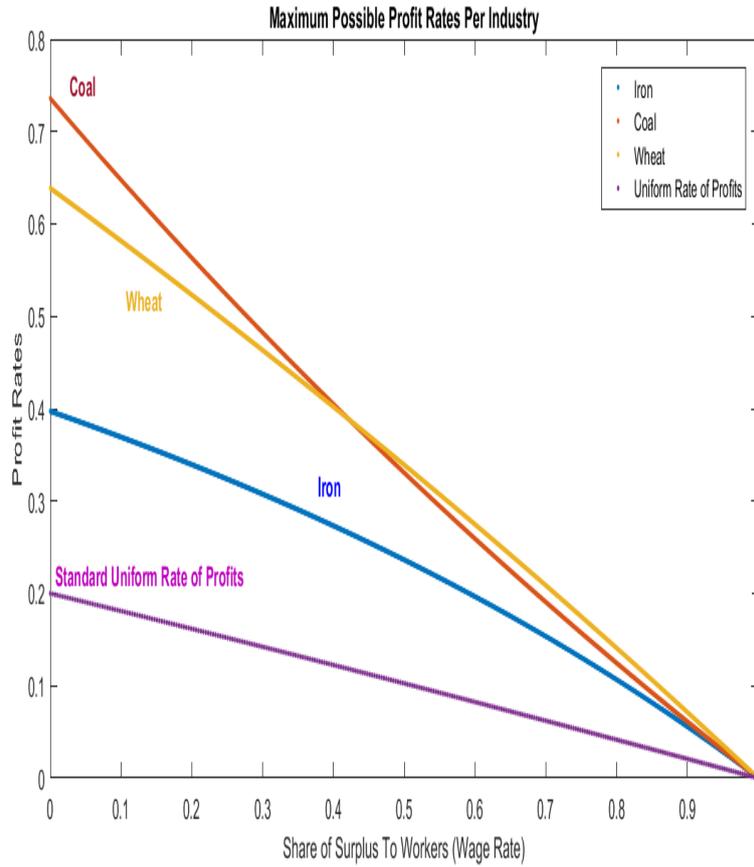


Fig. 2.2: **Prices that would allow Self-Replacing.** *Inside the triangles, for a given distribution to workers, there would be prices that would allow self reproduction of the system. Outside the triangle the replication is possible only if lending and borrowing is allowed. The triple  $\mathbf{A}$ ,  $\mathbf{L}$ ,  $\mathbf{b}$  used for this figure is reported above in 2.1 (Sraffa, 1960a, p.19). See also the related figure 2.1.*

are not the profit levels of the industries. This is my be counterintuitive with respect to the notion of free competition equilibrium. Capitalists investing in the coal and iron industry would have a lower share of the physical surplus (although the profit rate is the same across industries). The data of the example of table 2.1 is that relative to the share of surplus to workers equal to zero and the data of the example of table 2.2 is relative to the share of surplus to workers equal to 80%.

We have argued, with the aid of the examples developed in the previous sections, that a free competition solution could also imply a case where, for a given share to workers, the distribution in shares to industries is the same. Figure 2.5 reports the profit rates that would be associated with an even distribution in the shares. As it is clear from the figure the profit rates would surely not be uniform. Clearly the iron industry would get a higher share in the case in which profit rates are uniform, but would be associated with the lowest profit rate when there is uniformity among industries in the distribution in shares.

Some authors would or could argue that it is the average of the profit rates that would be important. This average could be related with the standard case of the uniform rate of profits. Figure 2.6 (upper figure) shows the great variety of profit rates that would be associated with the same average rate of profits for the case in which the share to workers is zero. The maximum rate of profits in the case in which we have uniform rate of



**Fig. 2.3: Maximum Individual Industry Profit Rate as a function of the Share of the Surplus to Workers.** *The lowest curve is the traditional wage-profit rate curve (with the axes exchanged). The other three curves are the maximum rate of profits per industry. Clearly, for example, if at a given share of the surplus to workers is associated the maximum rate of profits to the coal industry the rate of profits of the other industries the other two industries' rate of profits would be zero. Given two rates of profits the remaining rate of profits is uniquely determined. The triple  $\mathbf{A}, \mathbf{L}, \mathbf{b}$  used for this figure is reported above in 2.1 (Sraffa, 1960a, p.19).*

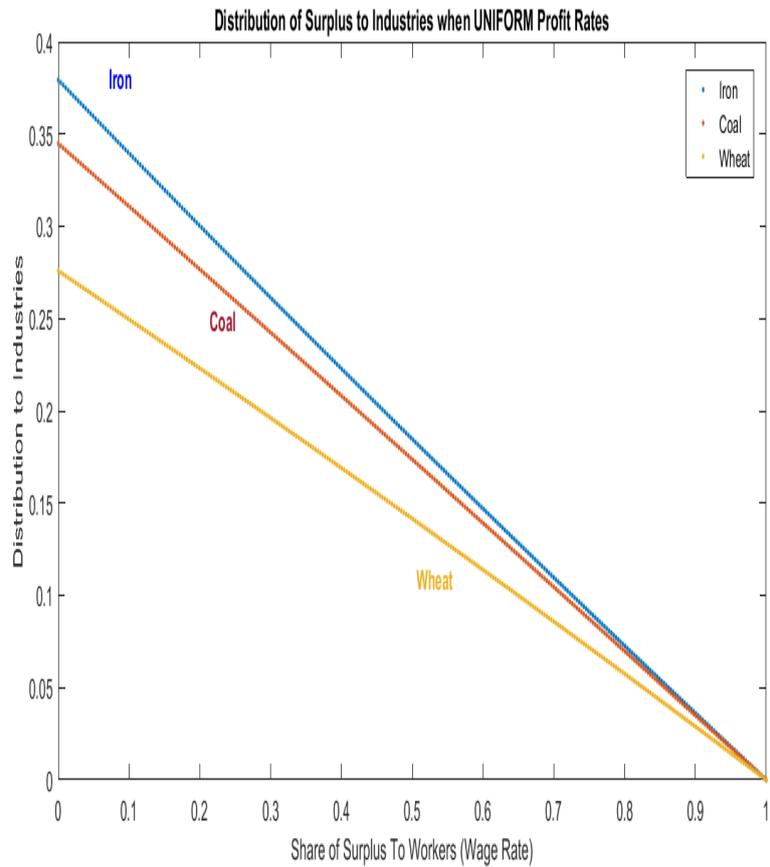


Fig. 2.4: **Distribution of Physical Surplus to industries when the rate of profits is uniform.** *The triple  $\mathbf{A}$ ,  $\mathbf{L}$ ,  $\mathbf{b}$  used for this figure is reported above in 2.1 (Sraffa, 1960a, p.19).*

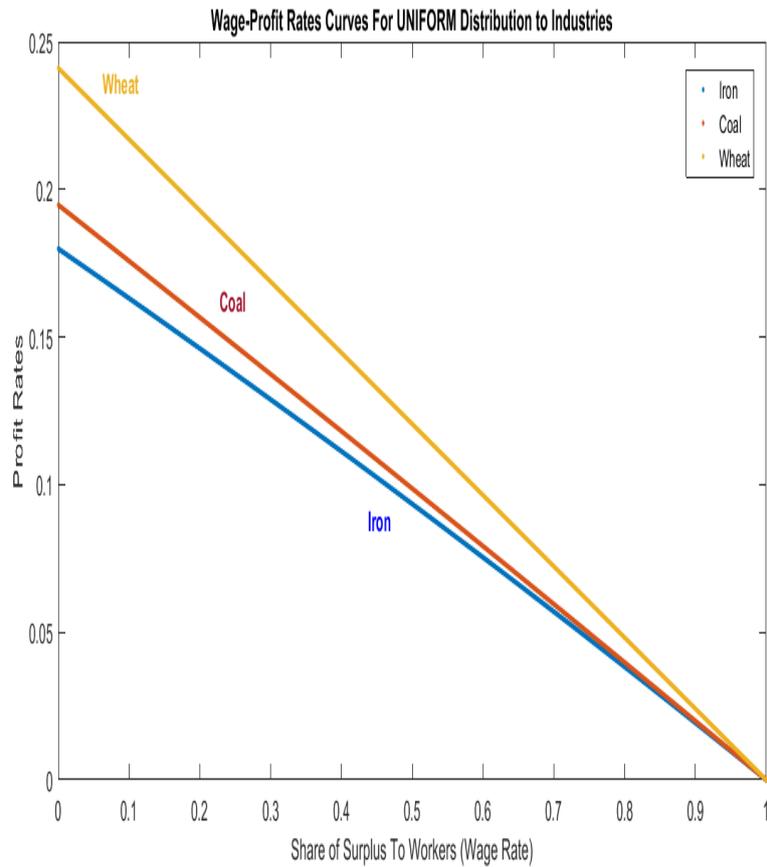


Fig. 2.5: **Industry Profit Rates** for the case of uniform distribution of the **Physical Surplus to industries**. The triple  $\mathbf{A}$ ,  $\mathbf{L}$ ,  $\mathbf{b}$  used for this figure is reported above in 2.1 (Sraffa, 1960a, p.19).

profits would be 0.20 (i.e 20%, see the lowest wage-profit curve of figure 2.3 and table 2.1). The figure shows that there would be a great variety of combinations of profit rates which would be associated with the same average rate of profits.

The lower figure of 2.6 shows the combinations of the individual rate of profits that would lead to the average rate of profits. A unique link between the average of rates of profits and distribution seems not to be possible.

### **3 Prices consistent with uniform rate of profits across industries a special case of non-uniform rates of profits prices.**

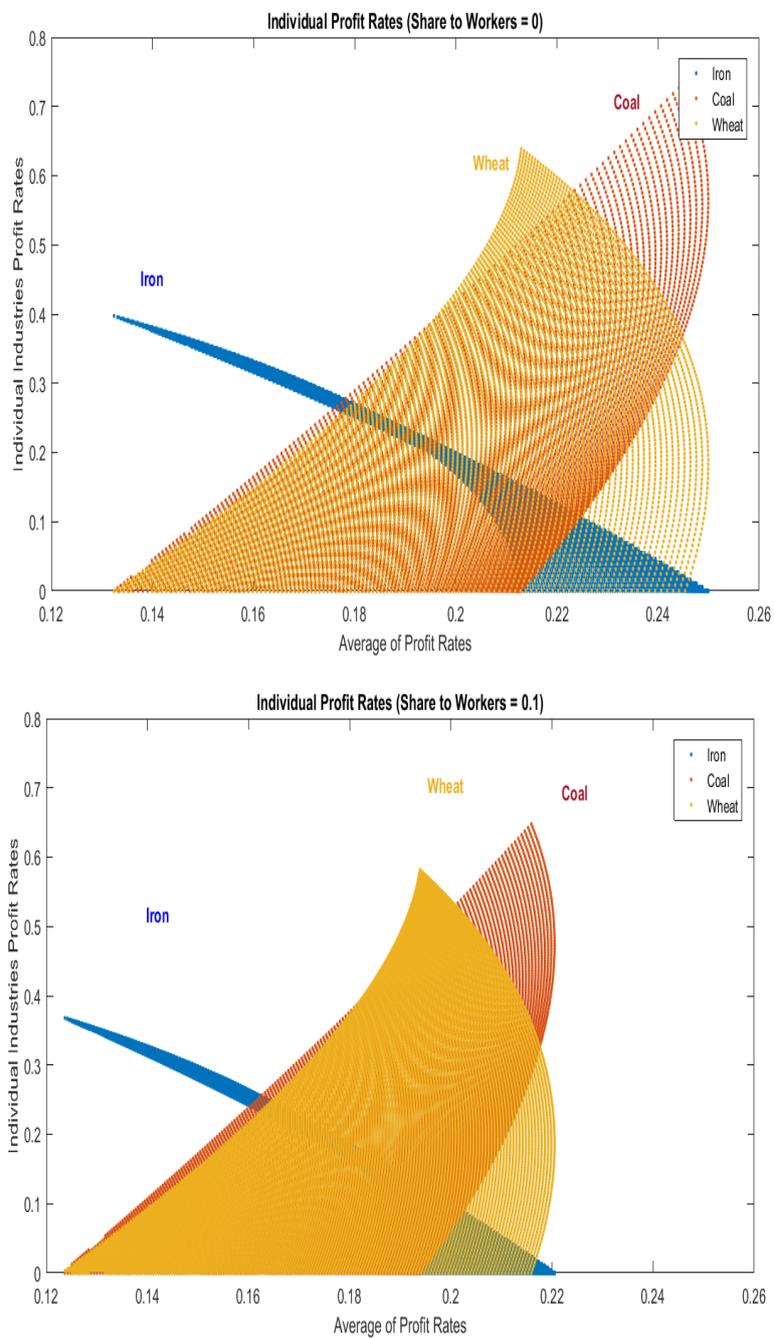
Many readers, independently from whether they are familiar or not with Sraffian themes or with the theories of the Classical Economists like Smith, Ricardo and Marx, might be uneasy with the idea that the profit rates may be non-uniform and yet the system could be in a self-replacing state.

As we have shown above (see in particular figure 2.4) to the uniform rate of profits solutions there would be associated different shares or profit levels to the individual industries. The classical *free competition* has as a logical starting point the search for the highest return of the investment which is not at all the same as the rate of return of the investment. If what we have shown above is correct (and we think it is) this aspect seems to have been overlooked.

This fact does not have serious implications on the conclusions to be derived or that are explicitly present in PCMC. As a matter of fact all the propositions made by Sraffa or derivable from PCMC are not dependent on whether the rates of profits are uniform across industries. One has to keep in mind that the distributions associated to the uniform rate of profits solutions belong to the set of all possible distributions that would allow the system to replicate. To each vector of profit rates  $\mathbf{r}$  and share of the surplus to workers ( $w$ ) there is associated one and only one price vector  $\mathbf{p}$  and hence just one distribution of the surplus vector  $\mathbf{d}$  and vice versa, i.e. to a given distribution in surplus vector  $\mathbf{d}$  there is associated one and only one vector of profit rates  $\mathbf{r}$ , share of the surplus to workers ( $w$ ), price vector  $\mathbf{p}$ . The uniform rate of profits vectors are just a subset of all possible vectors which would satisfy the self-replacing condition as defined above in eq.1.12.

#### **3.1 Non-uniform rates of profits do not change the major propositions and implications present in PCMC**

In our view none of the important proposition made under the assumption of the uniform rate of profits would be meaningless when we consider the



**Fig. 2.6: Combinations of different industry profit rates and average profit rate.**

*Upper graph Share to Workers = 0*

*Lower graph Share to Workers = 0.1.*

*The triple  $\mathbf{A}$ ,  $\mathbf{L}$ ,  $\mathbf{b}$  used for this figure is reported above in 2.1 (Sraffa, 1960a, p.19).*

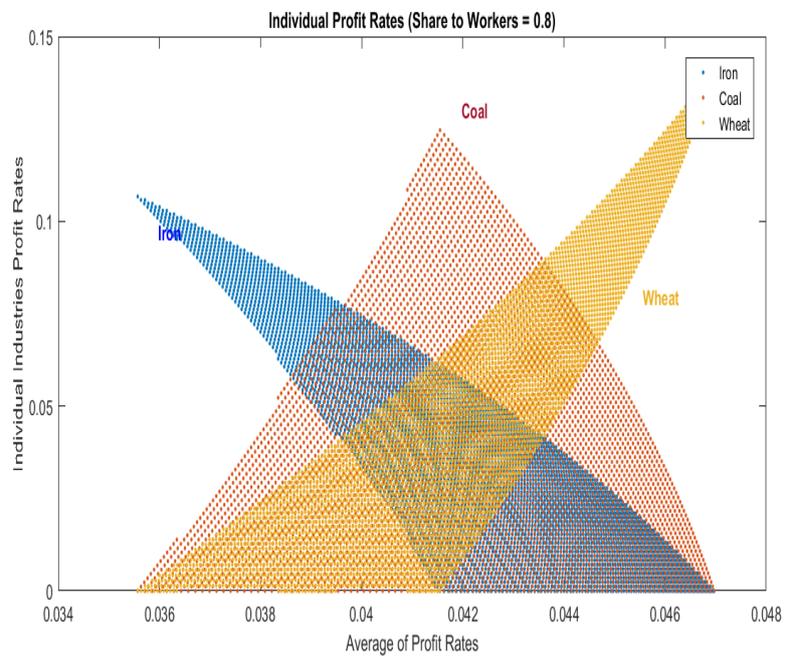


Fig. 2.7: Combinations of different industry profit rates and average profit rate. Share to Workers = 0.8. The triple  $\mathbf{A}$ ,  $\mathbf{L}$ ,  $\mathbf{b}$  used for this figure is reported above in 2.1 (Sraffa, 1960a, p.19).

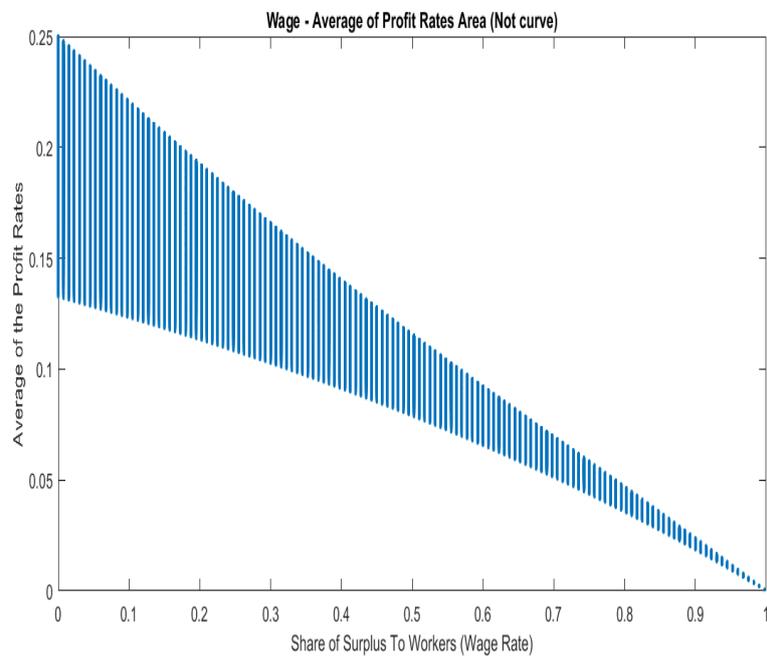


Fig. 2.8: **Combinations of different industry profit rates and average profit rate.** *The triple  $\mathbf{A}$ ,  $\mathbf{L}$ ,  $\mathbf{b}$  used for this figure is reported above in 2.1 (Sraffa, 1960a, p.19).*

more general case of non-uniform rates of profits. This is simply due the observation that the uniform rate of profits cases are just a subset of all the cases. Here there is no space to develop it further, but it is also the case that the *Prelude to a Critique of Economic Theory* holds and it is stronger when we study the self-replacing conditions having as logical departure the distributional vector, given which all the other values are derived.

For example the very important result or claim that the computed prices are independent of the level of activities does not depend on whether there is a uniform rate of profits  $r$  or a vector of profit rates  $\mathbf{r}$ . Therefore relative prices, like in the standard PCMC case, are not a function of the quantities or of level of the activities<sup>21</sup>. Therefore it can only be confirmed that prices depend on the set of methods and, most importantly, distribution.

Furthermore it can also be shown that the reduction to dated quantities of labour, (Sraffa, 1960a, Ch. VI, pp. 34–40) can be worked out with the same implications for the case of non-uniform rates of profits (see footnote 1, p.3).

It might be important to stress also that the substance of switching, re-switching and capital reversing results that were at the core the Cambridge Capital Controversy remains still valid and, if anything, is magnified when we consider non-uniform rates of profits (Pasinetti (1966), Garegnani (1966), Samuelson (1966)).

Moreover, it should be stressed that here the focus is on distribution. All those whose approach to economic theory is based on the importance of distribution – being determined by social and institutional factors and not exclusively by supply and demand – should see favorably the conclusions put forward in this paper. For example, the so called gravitational theory, (Gargnani (1976); Garegnani (1997)), and the debates centered on it (Sinha (2010, Ch. 4), does not depend in a crucial way on whether the scalar uniform rate of profits is replaced by a vector of rates of profits.

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<sup>21</sup>This is also known as the *Non-Substitution Theorem*. Given any productive re-proportioning  $\mathbf{X}$  of the the original triple  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{L}$ , it is the case that the prices are not dependent on this re-proportioning. Where  $\mathbf{X}$  is a semipositive diagonal matrix, which represents the intensity of the utilization of the methods used (the activity levels). The system is defined as being productive for all those cases in which  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{X}$  are such that  $\mathbf{e}'(\mathbf{XB} - \mathbf{XA}) \geq 0$ , where  $\mathbf{e}$  is the unit or summation vector. Where  $\mathbf{X}$  is a semipositive diagonal matrix, which represents the intensity of the utilization of the methods used (the activity levels).

$$\begin{aligned} (\mathbf{I} + \mathbf{R})\mathbf{X}\mathbf{A}\mathbf{p} + \mathbf{X}\mathbf{L}w &= \mathbf{X}\mathbf{B}\mathbf{p} \\ \mathbf{p} &= [\mathbf{X}\mathbf{B} - (\mathbf{I} + \mathbf{R})\mathbf{X}\mathbf{A}]^{-1}\mathbf{X}\mathbf{L}w \\ \mathbf{p} &= [\mathbf{B} - (\mathbf{I} + \mathbf{R})\mathbf{A}]^{-1}\mathbf{X}^{-1}\mathbf{X}\mathbf{L}w \\ \mathbf{p} &= [\mathbf{B} - (\mathbf{I} + \mathbf{R})\mathbf{A}]^{-1}\mathbf{L}w \end{aligned}$$

And this is equivalent with eq. 1.27

### 3.2 Natural or *necessary* prices

The prices computed here are the prices that would be necessary for the system to be in a self-replacing state. These are not at all market prices. What is the relation with these prices w.r.t. *natural prices, production prices*? In the terminological note of sec. 7, pp.8-9, of PCMC Sraffa explains the use of the terms prices and value in relation with the use of terms like *cost of production* and *capital*. His need for clarification of these terms was because, as he writes, he wanted to avoid misunderstanding. He wrote:

... A less one-sided description than costs of production seems therefore required. Such classical terms as ‘necessary price’, ‘natural price’ or ‘price of production’ would meet the case, but value and price have been preferred as being shorter and in the present context (which contains no market prices) no more ambiguous. It may be added that not only in this case but *in general* the use of the term ‘cost of production’ has been avoided in this work, as well as the term ‘capital’ in its quantitative connotation, at the cost of some tiresome circumlocution. This is because these terms have come to be inseparably linked with the supposition that they stand for quantities that can be measured independently of, and prior to, the determination of the prices of the product. (Whitness the ‘real cost’ of Marshall and the ‘quantity of capital’ which is implied in the marginal productivity theory.) Since to achieve freedom from such presuppositions has been one of the aim of this work, avoidance of the term seemed the only way of not prejudicing the issue. (Sraffa, 1960a, p.9)

We think that the method for the computation of the prices that we have followed here is consistent with Sraffa’s terminological note.

The fact that Sraffa had chosen in the previous page to discuss prices that would be equalizing revenues and expenditures of his basic system with the additional assumption of uniform rate of profits is not relevant for the content of his terminological note. A careful inspection on his terminological note should convince any reader that what he asserts does not depend on whether the rates of profits are uniform across industries or not<sup>22</sup>.

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<sup>22</sup>To be as clear as possible, to avoid misunderstanding, let me be pedantic on this point. At page 6 of PCMC Sraffa writes the system of equations where the rate of profits is uniform. If he had written these equation in the following way

$$\begin{aligned} (A_a p_a + B_a b_b + \dots + K_a p_k)(1 + r_a) &= A p_a \\ (A_b p_a + B_b b_b + \dots + K_b p_k)(1 + r_b) &= B p_b \\ &\dots\dots\dots \\ (A_k p_a + B_k b_b + \dots + K_k p_k)(1 + r_k) &= K p_k \end{aligned}$$

We believe that we follow Sraffa's proposed method of analysis quite closely, the only different being that we operate with a vector of profit rates, in stead of just one value. For the rest everything is from the point of view of content very similar, if not the same. Sraffa's terminological note can be applied for the prices defined in this paper.

### 3.3 Are Sraffian Schemes a special case of Walrasian General Equilibrium, or is Walrasian General Equilibrium a special case of Sraffian Schemes?

It has been said by several authors that the Sraffian Schemes (eq. 1.13, 10 may be interpreted as budget constraints of Walrasian General Equilibrium (Hahn (1982); Mandler (1999))).

Hahn (1982) has claimed that Sraffian Schemes are just a very special case of the more General Walrasian System of equations or equilibrium. The uniform rate of profits condition is thought to be a requirement of Sraffa's methodology, this being true it is claim that Sraffian Schemes would be a special case of Walrasian General Equilibrium simply because it allows for different profit rates.

The equilibrium prices in a modern Walrasian General Equilibrium are those that would a) assure *market clearing* and b) such that all agents maximize their utility (or profits) functions (subject to budget constraints).

In this paper we make a case for *generalized* Sraffian Schemes where the rates of profits are not uniform. Our prices, which depend on distribution, are all, "*market clearing*" prices. Therefore the argument that Sraffian Schemes are a special case of Walrasian General Equilibrium does not hold any longer. It is actually the contrary. The Walrasian General Equilibrium is at most, if anything, a special case of generalized Sraffian schemes.

## 4 Conclusions

We have investigated the conditions that would allow the economic system to be in a self-replacing state. From the analysis developed here we can conclude several thing.

- a) **Non-uniform rates of profits and Self-Replacing Condition.** The economic systems when described with the observed quantities as in the Sraffian schemes would be in a self-replacing state for cases in which the profit rates are non-uniform as well as when they are uniform. If we interpret the self-replacing prices as *natural* prices and the wage rate

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This system, with change in notation, is the system of eq. 1.13 or 1.14 for the case in which the wage rate  $w$  is zero. Conceptually, to say that  $r$  is our independent variable or unknown, or to say that our independent variable is a vector of profit rates, is substantially the same thing.

as a *natural* share of the output to workers, there are many individual industry profit rates associated to them. The uniform rate of profits distribution is a special case, but not a necessary one for the self-replacing condition.

- b) **Uniqueness I. From prices to profit rates and distribution.** Once self-replacing prices and the wage rate are given, there is a unique vector of profit rates associated to it and a unique distribution associated to it.
- c) **Uniqueness II. From distribution to prices, profit rates and distribution.** For any given or observable (functional or personal) distribution of the surplus (i.e. the Net National Product) there is only one unique set of prices, rates of profits, and wage rate that would allow a self-replacing state.
- d) **Problematic I. Uniform Rates of profits imply non uniform profit levels.** When the rate of profits is uniform across industries, the profit levels (i.e. the shares of the surplus or industry incomes) are in general not uniform, see fig. 2.4. This observation is somewhat problematic because to the same rate of returns of an investment would correspond different returns or shares. If we interpret the producers described in our exercise as capitalists investing in different industries one could agree that they would want to invest the value of their capital (i.e. the physical quantities sold at the prices found – or determined – during the market operations). But the profit levels associated with the prices that would be associated with uniform rates of profits are different, hence a uniform rate of profits is not an indication of a system that has settled to a free competition equilibrium.
- e) **Problematic II. Uniform Rates of profits imply non uniform profit levels.** When distribution in shares among the industries (i.e., the profits) are uniform (see fig. 2.5) the rates of profits are not. This is also problematic in the same way, with reverse logical direction, as in point d) above. Uniform profit levels are associated with non-uniform profits.
- f) **Sraffian schemes are not just a special case of Walrasian General Equilibrium.** The argument that Sraffian schemes are special case of modern Walrasian General Equilibrium Hahn (1982) because of the uniform rate of profits assumption does not, given that the non-uniform rates of profits condition is not a necessary condition, hold. The contrary can in fact be claimed: modern General Walrasian Equilibrium is at most a sub-case of Sraffian schemes.
- g) **Stronger support on the importance of distribution.** An important result which is presented here is that once a functional or personal

distribution of the surplus is given the self-replacing prices, vector of profit rates and the wage rate are uniquely determined. Sraffa himself in PCMC and authors that followed him have strongly emphasized the fact that at different (uniform) rate of profits there would be associated a different distribution. This profit rate (or distribution), although necessary to determine the necessary prices to allow for self-replacing would not be determined inside the system, but would require other *outside* elements. This is welcomed as an element of realism. I obviously agree with this interpretation. In this paper (see also eqs.1.19 and 1.20) it is claimed that given a distribution vector we have a unique price vectors and a unique non-uniform set rates of profits. From the logical point of view the importance of distribution and the necessity to explain its determinants does not depend on whether the rates of profits are equal or not.

The above is a list of results or claims that can be made when one removes the condition of uniform rate of profits. Several lines of research open up. One of them is the potential link between Sraffian schemes and Keynesian theory of demand. Pasinetti (1981, 1993) has made several attempts to link the long-run (i.e. with the assumption of a uniform rate of profits) properties of Sraffian schemes with the short run Keynesian demand theory<sup>23</sup>.

It is clear from recent debates and contributions (Arena, 2013; Lavoie, 2010) that Sraffa's contribution has been relegated to the long-run period or to the world of production prices where there is little or no role for demand and/or for money or credit. I believe that it has not to be so. In this paper an argument in support of a non-uniform rate of profits interpretation of Sraffian schemes has been presented.

In the case that this article passes the scrutiny of the academia, the results presented here could be used as a starting point for empirical applications to economic policy (both for the short run as well for the long run). But this would be the task of next paper.

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<sup>23</sup>A discussion in the direction of bridging Sraffa's and Keynes has been made by Minsky (1990). Also in this case, as the title of the contribution clearly indicates, *Sraffa and Keynes: Effective Demand in the Long Run.*, Sraffa's contribution is relegated, due to the assumption of uniform rate of profits, to the long-run.

## A Actual and equivalent distribution of the surplus

The physical surplus  $\mathbf{S}$  is obviously distributed among the members of the society (producers and workers). We can arrange the possible or observable per capita distribution of the physical surplus in a matrix,  $\bar{\mathbf{H}}$ , whose rows are the *actual* or *observed* individual physical distribution of the surplus<sup>24</sup>.

$$\bar{\mathbf{H}} = \begin{bmatrix} \bar{\mathbf{h}}_1 \\ \bar{\mathbf{h}}_2 \\ \vdots \\ \bar{\mathbf{h}}_i \\ \vdots \\ \bar{\mathbf{h}}_n \\ \bar{\mathbf{h}}_w \end{bmatrix} = \begin{bmatrix} \bar{h}_1^1 & \bar{h}_1^2 & \dots & \bar{h}_1^j & \dots & \bar{h}_1^n \\ \bar{h}_2^1 & \bar{h}_2^2 & \dots & \bar{h}_2^j & \dots & \bar{h}_2^n \\ \vdots & \vdots & & \vdots & & \vdots \\ \bar{h}_i^1 & \bar{h}_i^2 & \dots & \bar{h}_i^j & \dots & \bar{h}_i^n \\ \vdots & \vdots & & \vdots & & \vdots \\ \bar{h}_n^1 & \bar{h}_n^2 & \dots & \bar{h}_n^j & \dots & \bar{h}_n^n \\ \bar{h}_w^1 & \bar{h}_w^2 & \dots & \bar{h}_w^j & \dots & \bar{h}_w^n \end{bmatrix} \quad (\text{A.1})$$

$\bar{\mathbf{H}}$  is an  $n \times n+1$  matrix. The first  $n$  rows are the distribution of the National Surplus to the  $n$  industries (or producers) and the last, the  $n^{\text{th}}+1$  row, is the distribution of the surplus that goes to the workers. Clearly the column sum of the matrix has to give the surplus vector  $\mathbf{S}^T$ . In the situation in which we assume that commodity prices are uniform we have that the purchasing power necessary to buy a consumption bundle  $\bar{\mathbf{h}}_i$  is given by  $\bar{\mathbf{h}}_i \mathbf{p}$  (where  $\mathbf{p}$  is the  $n \times 1$  commodities price vector). In value terms this is also equal to the values share  $d_i$  of the total Net National Product so that  $\bar{\mathbf{h}}_i \mathbf{p} \equiv d_i \mathbf{S}^T \mathbf{p}$ .

Among all possible physical distributions of the surplus there is a subset which will be particularly useful for the analysis developed here. A matrix belonging to this subset would be the following

$$\mathbf{H} = \mathbf{d} \mathbf{S}^T \quad (\text{A.2})$$

where  $\mathbf{d}$  is the Net National Product distribution vector with  $\sum_i^{n+1} d_i = 1$ , i.e., it is the share of the surplus distributed between producers and workers<sup>25</sup>.

<sup>24</sup>In order to avoid unnecessary complications we consider here distribution among the industry and the workers as a whole. Clearly the number of rows of  $\bar{\mathbf{H}}$  could be as many as the individuals forming the society

<sup>25</sup>The dimension of the vector  $\mathbf{d}$  could be very large. In the case in which we extend the analysis considering the distribution of the surplus to all the producers contributing to the industries and of all the workers the dimension of  $\mathbf{d}$  would be the total amount of agents (population) belonging in the system. Just as an example, if the number of producers were 13 per industry and the industries were 7, while the workers were 3709 the number of columns of  $\mathbf{H}$  or the dimension  $\mathbf{d}$  would be  $3800 = 13 \times 7 + 3709$ . The trivial, but also very important observation, is that the sum by rows of  $\mathbf{H}$  would give the total amount of Surplus to be distributed. In the case of the Physical Surplus to be distributed would be the  $\mathbf{S}$  vector, which has as element the surplus of the 7 commodities used by the system.

A very important characteristic of the distribution  $\mathbf{H}$  is that the composite physical distribution vector associated to each agent is a fraction of the total physical surplus generated. If this were the case the comparison between the consumption of two agents, say agent  $i$  and agent  $j$ , could be done simply by comparing the two composite bundles<sup>26</sup>.

We have an important equivalence relation which is given by the following:

$$\begin{array}{c}
 \text{Shares of Surplus} \\
 \underbrace{\begin{bmatrix} d_1 \mathbf{S}^T \mathbf{p} \\ d_2 \mathbf{S}^T \mathbf{p} \\ \vdots \\ d_i \mathbf{S}^T \mathbf{p} \\ \vdots \\ d_n \mathbf{S}^T \mathbf{p} \\ d_w \mathbf{S}^T \mathbf{p} \end{bmatrix}}_{\mathbf{dS}^T \mathbf{p}} \\
 \text{Effective Demands} \\
 \underbrace{\begin{bmatrix} \bar{\mathbf{h}}_1 \mathbf{p} \\ \bar{\mathbf{h}}_2 \mathbf{p} \\ \vdots \\ \bar{\mathbf{h}}_i \mathbf{p} \\ \vdots \\ \bar{\mathbf{h}}_n \mathbf{p} \\ \bar{\mathbf{h}}_w \mathbf{p} \end{bmatrix}}_{\bar{\mathbf{H}} \mathbf{p}} \\
 \text{Effective Demands} \\
 \text{Equivalents} \\
 \underbrace{\begin{bmatrix} \mathbf{h}_1 \mathbf{p} \\ \mathbf{h}_2 \mathbf{p} \\ \vdots \\ \mathbf{h}_i \mathbf{p} \\ \vdots \\ \mathbf{h}_n \mathbf{p} \\ \mathbf{h}_w \mathbf{p} \end{bmatrix}}_{\mathbf{H} \mathbf{p}}
 \end{array}
 =
 \begin{array}{c}
 \text{Effective Demands} \\
 \underbrace{\begin{bmatrix} \bar{\mathbf{h}}_1 \mathbf{p} \\ \bar{\mathbf{h}}_2 \mathbf{p} \\ \vdots \\ \bar{\mathbf{h}}_i \mathbf{p} \\ \vdots \\ \bar{\mathbf{h}}_n \mathbf{p} \\ \bar{\mathbf{h}}_w \mathbf{p} \end{bmatrix}}_{\bar{\mathbf{H}} \mathbf{p}} \\
 \text{Effective Demands} \\
 \text{Equivalents} \\
 \underbrace{\begin{bmatrix} \mathbf{h}_1 \mathbf{p} \\ \mathbf{h}_2 \mathbf{p} \\ \vdots \\ \mathbf{h}_i \mathbf{p} \\ \vdots \\ \mathbf{h}_n \mathbf{p} \\ \mathbf{h}_w \mathbf{p} \end{bmatrix}}_{\mathbf{H} \mathbf{p}}
 \end{array}
 \quad (\text{A.4})$$

Obviously if  $\mathbf{p}$  is given also the price ratios are given. Hence to operate on the basis of the presumed actual physical distribution  $\bar{\mathbf{H}}$  is equivalent

<sup>26</sup>The comparison of the consumption or surplus bundles of two agents may be problematic because the bundles of goods may have different proportions,  $\bar{\mathbf{h}}_i \not\cong \bar{\mathbf{h}}_j$ . Hence the ratios of the individual goods composing the bundles would most likely be non uniform. That is  $\frac{\bar{h}_i^1}{\bar{h}_j^1} \neq \frac{\bar{h}_i^2}{\bar{h}_j^2} \neq \dots \neq \frac{\bar{h}_i^n}{\bar{h}_j^n}$ . Once the prices are given we can compare the values of their bundles and we can compare the value of their bundles with respect to the value of the total surplus. The share in value of the surplus for agent  $i$  would be  $d_i = \frac{\bar{\mathbf{h}}_i \mathbf{p}}{\mathbf{S}^T \mathbf{p}}$  and for agent  $j$  would be  $d_j = \frac{\bar{\mathbf{h}}_j \mathbf{p}}{\mathbf{S}^T \mathbf{p}}$ . Therefore we can compare the values of these bundles by comparing their shares of the total surplus:  $\frac{d_i}{d_j} = \frac{\bar{\mathbf{h}}_i \mathbf{p}}{\bar{\mathbf{h}}_j \mathbf{p}}$ . If we have that  $d_i > d_j$  we can say that the value of the consumption output of agent  $i$  is greater than the value of the consumption bundle of agent  $j$ , but we cannot conclude in all cases that agent  $i$  has a consumption in physical terms which is greater than  $j$  because agent  $i$  may consume more of one good and less of another with respect to agent  $j$ .

Quite different would be the case in which the distribution was as in eq.A.2,  $\mathbf{H} = \mathbf{dS}^T$ . In this case we have that

$$\frac{h_i^1}{\bar{h}_j^1} = \frac{h_i^2}{\bar{h}_j^2} = \dots = \frac{h_i^n}{\bar{h}_j^n} = \frac{d_i}{d_j} = \frac{d_i}{d_j} = \frac{\mathbf{h}_i \mathbf{p}}{\bar{\mathbf{h}}_j \mathbf{p}} = \frac{\bar{\mathbf{h}}_i \mathbf{p}}{\bar{\mathbf{h}}_j \mathbf{p}} \quad (\text{A.3})$$

For the whole system we have the following:  $\bar{\mathbf{H}} \mathbf{p} = \mathbf{H} \mathbf{p} = \mathbf{dS}^T \mathbf{p}$ . These different distributions are equivalent in the sense that they belong to the same *budget constraint line* in the same meaning which is normally understood also in standard microeconomics (see xxxx). Therefore, for clarity and analytical purposes we will consider the distribution  $\mathbf{H}$  because it is equivalent, for the reasons just explained to the observed or actual distribution  $\bar{\mathbf{H}}$

with  $\mathbf{H}$  (see also footnote 26, p.34). In the sequel we will work with matrix  $\mathbf{H}$  which is perfectly exchangeable with  $\bar{\mathbf{H}}$ <sup>27</sup>.

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<sup>27</sup>The notion of equivalence is used here in a very precise meaning.  $\mathbf{H}$  is equivalent to  $\bar{\mathbf{H}}$  if the propositions or computations do not depend on whether  $\mathbf{H}$  is replaced by  $\bar{\mathbf{H}}$  and viceversa. Some would say that  $\mathbf{H}$  is *exchangeable* with  $\bar{\mathbf{H}}$

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