STRUCTURAL FLAWS IN THE AXIOMATIC APPROACH TO POST KEYNESIANISM

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Abstract: The axiomatic approach has long been a central theme in US discussion of Post Keynesianism. Axioms constitute its foundations in at least three areas – its conception of an axiom, the manner in which it deploys axioms to characterise differences between Post Keynesianism and orthodoxy, and the key role ascribed to axioms in specifying general theory-special case relationships. This paper argues that the axiomatic approach is fundamentally flawed in being grounded on internal logical inconsistencies, textual inconsistencies in relation to the *General Theory* and the *Treatise of Probability*, and unsupportable conceptualisations. Three general conclusions are drawn. The axiomatic approach is incapable of providing an acceptable framework for interpreting the *General Theory*, it is unable to derive orthodoxy as a special case of Keynes's theory, and it is unsupportable as an adequate foundation for Post Keynesianism.

Keywords: Post Keynesian economics, axiomatic approach, general theory-special case relationships.

JEL classification: B31, B41, E00, E12.

The axiomatic approach (hereafter AA) is an accepted and influential strand in Post Keynesian thought, particularly in the US but also in the UK and elsewhere. First advanced by Paul Davidson in the mid-1980s and extended in the mid-1990s, it has been a prominent element in contributions by himself and his followers in the *Journal of Post Keynesian Economics* and other outlets. However, it has attained this status more by persistent assertion, repetition and advocacy than by emerging intact from comprehensive scrutinies of internal coherence and analytical soundness. Although deeper exploration of some elements in its architecture has recently been initiated,[1] this paper focuses on the entire AA framework. The discussion is necessarily lengthy because AA is a large theoretical construction with multiple foundations, inadequately explained statements, several conflations, and internal problems buried deep in its foundations.

Four propositions advanced or implied by AA are central in the present context.
1. The concept of an axiom – axioms represent universal truths, or beliefs in universal truths.
2. Fundamental axiom sets – orthodox economics and the *General Theory* (hereafter *GT*) repose on directly opposed triple axiom sets: ergodicity, money neutrality and gross substitution in relation to orthodoxy, and non-ergodicity, non-money neutrality and non-gross substitution in relation to the *GT*.[2]
3. General theories and special cases – their differentiation is based on the number of restrictive axioms, the former having fewer than the latter.
4. An epistemological precondition on rational behaviour – rational decision-making is ontology-dependent, such that rational agents must know whether they inhabit ergodic or non-ergodic states before engaging in economic activity.

This ensemble is marred by three major flaws: destructive logical inconsistencies, serious conceptual flaws, and remarkable textual flaws in relation to the *GT* and the *Treatise on Probability* (hereafter *TP*). All are important, but the first is most critical. Here AA is beset by four internal inconsistencies: it is incapable of deriving orthodoxy as a special case of the *GT*; the perfect knowledge required by its rationality precondition conflicts with its emphasis on uncertainty; its concept of an axiom clashes with the approach used by both Post Keynesian and orthodox economists; and the open-endedness of non-ergodicity cannot rescue it from the first inconsistency.[3]

These problems render it structurally flawed as an interpretation of Keynes's mature economics. The flaws have remained unexposed due to acceptance on trust or authority by enthusiasts, an absence of deeper scrutiny by doubters, and a lack of interest by the uncommitted.

The AA concept of an axiom

Our point of departure is AA's conception of axioms which are given particular meanings that connect them to universal truth in both strong and weak forms. According to Davidson, an axiom either *is*, or *is believed to be*, a universal truth.

An axiom is defined as a universal truth that need not be proved. (Davidson 2009c p.6)

An axiom is defined as 'a statement universally accepted as true … a statement that needs no proof because its truth is obvious'. (Davidson 2002 p.41; also 2007 pp.27, 36, 1994 p.15)

If any axiom is used as the basis for developing a theory, then the theorist who invokes this theory is implicitly claiming that that underlying axiom is a statement that he/she believes must be universally accepted as true. An axiom is a statement that needs no proof. (Davidson 2015a, pp.6-7; also 2009b p.30)

This contrasts starkly with the *GT* where Keynes repeatedly uses the terms 'assumptions' and 'postulates', and avoids the term 'axiom' except when used in other fields such as Euclidean geometry. And he does not associate assumptions or postulates with universal truth in any way. Like many economists, he used assumptions and postulates in their normal meaning as starting points for reasoning, or statements acting as foundations for deductive systems, without any reference to universal truth. Even while searching for as much realism as possible and deploying assumptions taken to be true under a wide variety of conditions, he never declared their universal truth. This includes his consumption function where Hodgson's (2001 pp.31, 221-2) claim that 'Keynes fell back on…supposedly universal 'psychological laws'' is mistaken. Careful reading shows that Keynes enveloped this function in qualifications. Immediately after labelling it a 'fundamental psychological law', he added that it was one 'upon which we are entitled to depend with great confidence', where great rather than complete confidence distances it from universal truth. He also described it as a 'general' statement operating 'as a rule' or under 'normal conditions', one taking income to be the 'principal' but not the only determinant of consumption, and one depending partly on 'social practices and institutions which are…not unalterable'. It is thus a proposition assumed to be sufficiently reliable under a wide variety of circumstances as to serve as a good starting point for realistic economic theorising − in short, a broad, non-universal truth suitable for theorising in social science and capable of being modified or replaced should circumstances turn out to be non-normal. The idea of a necessary connection between assumptions and universal truth is nowhere to be found in the *GT*.[4]

Also important here is AA's constant conflation of assumptions (in the normal general sense) with axioms (in the special AA sense). Whenever Keynes uses 'assumption', AA standardly substitutes 'axiom' as a synonym.[5]

AA's oppositions between orthodoxy and the *GT*

Here AA postulates two contrasting axiom sets, derived from a simple syllogism.

Orthodox economics is based on the three axioms of ergodicity, money neutrality and gross substitution.
Keynes rejected orthodox economics.
Hence he rejected the three orthodox axioms.
Therefore the *GT* is based on the three axioms of non-ergodicity, non-money neutrality and non-gross substitution.

Simple oppositionalism generates this manner of stating the fundamental differences between orthodoxy and the *GT*. AA starts with the three propositions viewed as the foundational orthodox axioms and moves, by simple negation, to the three propositions asserted to be the foundational axioms of Keynes and Post Keynesianism. As Davidson (2006a p.187) puts it:

The classical axioms that Keynes threw out in his revolutionary general analysis were (1) the neutrality of money axiom, (2) the gross substitution axiom, and (3) the axiom of an ergodic economic world.[6]

In moving from orthodoxy to the *GT*, Davidson simply adds the prefix, 'non-', to the names of the rejected axioms so as to characterise the axioms that Keynes supposedly adopted.[7] What this prefix signifies is the *absence* of the referents of the orthodox axioms. For example, Davidson (1988 p.332) associates uncertainty with 'the existence of nonergodic processes' *and* 'the absence of…ergodic processes'. The terms, non-ergodicity, non-money

neutrality and non-gross substitution, thus refer to all that lies outside, or is other than, the content of the orthodox axioms.

Although AA presents all three orthodox propositions as axioms, and as typically having equal significance, there is a sense in which the ergodic proposition is the more fundamental and also a proper axiom. It relates to assumptions concerning knowledge and uncertainty, these subsequently leading to the issues of money neutrality and inter-temporal substitution. And while ergodicity has sense as an axiom, money neutrality and gross substitution have greater resemblances to conclusions. This point is not critical in that logic transfers ideas embodied in axioms through to conclusions, but it does indicate a certain looseness in the handling of assumptions and conclusions, one that later plays an important role in generating logical inconsistencies. Each opposed axiom pair is now examined.

The ergodic and non-ergodic axioms

These provide the ontological foundations which AA regards as essential to economic analysis. Only two states of reality are admitted, ergodicity and non-ergodicity. Ergodic reality has a single invariant underlying numerical probability distribution generating outcomes over infinite time and space. Because every event is generated probabilistically, it is associated with *actuarial* certainty, which is inaccurately taken as identical to certainty. By contrast, non-ergodic reality is a state in which no such probability distribution exists. Its outcomes are driven by whatever forces are present other than a unique, universal numerical probability distribution. In AA, only this state is associated with 'uncertainty', taken in the sense of uncertainties entirely disconnected from probabilities.   Here AA adopts Knight's formulation of the risk-uncertainty distinction which entails a probability-uncertainty divorce, and the relative frequency theory of probability as against other interpretations (including Keynes's own logical theory of probability).

In relation to rational decision-making, the contrast is as follows. Orthodox economic agents inhabiting ergodic reality use the unique probability distribution in all their optimisations; not to do so would lead to economic failure and extinction. But in Keynes's economics which assumes non-ergodicity, the situation is otherwise − a single invariant probability distribution is absent, uncertainty rules, and rational agents must turn to alternative procedures. These are exemplified by uncertainty-loving attitudes such as 'Damn the torpedos, full steam ahead', or creative plans to self-construct the future through innovation or new ventures. Such kinds of behaviour facilitate the survival and success of rational agents in this environment.

Note that the ontology-dependence of rationality is essential. In each ontological state, economic survival and prosperity requires the alignment of decision procedures with the relevant underlying reality. Ignoring the probability distribution in ergodic environments will lead to failure, just as implementing probabilistic procedures will in non-ergodic environments where objective probabilities (of the required kind) are non-existent. Note also that agents must know (or know how to rapidly determine) which ontology exists *in advance of* making any rational decisions concerning economic activity.[8]

The money neutrality and non-money neutrality axioms

Money neutrality (the 'axiom of reals' or the 'classical dichotomy') is the orthodox view of causal independence between money and real variables such that money-real interactions never occur in the long run. Real variables determine real variables and money variables determine money variables, so that doubling the money supply merely doubles money prices and wages with no changes in relative prices, real output or employment. More significantly, money neutrality necessarily occurs in orthodox real-exchange models where money in any genuine sense is absent, so that the possibility of money determining anything is excluded.

Non-money neutrality is the opposing proposition that causal interactions can occur in the long run. Joint determination exists, with real variables determined by other real variables and money variables combined. Doubling the money supply produces, in general, long run changes in real variables (output and employment) as well as in money variables (money prices and wages). Post Keynesianism maintains non-money neutrality in all runs (as does orthodox 'rational expectations theory'), but it is the long run that differentiates all variants of orthodoxy from Post Keynesianism.[9]

The gross substitution and non-gross substitution axioms

These concern the different substitution possibilities in orthodox real-exchange economies and Keynes's money-using economy. The former are comprised solely of 'reproducibles, or commodities of variable supply that are produced and exchanged over time and in the absence of money and money-based assets. Substitution is always between such commodities, and never between commodities and monetary items. Even if one commodity is chosen as numeraire and called 'money', it is not money in any genuine sense, merely a unit of account that never changes hands as genuine mediums of exchange must do.[10] In this economy, gross substitution is a *universal* proposition applying to all exchanges. Rises in the price of any commodity lead to falls in its demand (movements along its demand curve), and to increases in demand for all its substitutes (shifts in their demand curves). Given maximising agents, any value not expended on that commodity (and possibly its complements) is fully expended on its substitutes (and possibly their complements), so maintaining the full employment of all resources. This value never lies 'idle' or in monetary 'resting places', but is always spent on other commodities due to omnipresent optimisation.

In Keynes's monetary production economy, where money is always present as a medium of exchange, store of value and unit of account, the gross substitution axiom does not hold universally. If the price of a reproducible commodity rises and its demand falls, any money saved by the reduced expenditure faces a wider range of options. The rational agent may save all or part of it in the form of money (currency, bank deposits), other liquid financial assets (bonds, equities), or collectibles in fixed supply (rare art works). Some or all may also be spent on other reproducibles. But a range of options exists, an important subset of which concerns 'non-reproducibles' as stores of value which provide 'resting places' for money balances not completely spent in ways directly increasing production or employment. There is no guarantee that aggregate production and employment will remain the same as before.[11]

The presence/absence opposition

All three axiom pairs have a common form, an opposition based on the presence or absence of a particular thing:
(i) Ergodicity means the presence of a *unique underlying probability distribution* driving all outcomes across time and space. Non-ergodicity means the complete absence of such a distribution.
(ii) Money neutrality means the *complete absence* of money-real interactions in the long run, while non-money neutrality means their presence.
(iii) Gross substitution means the *complete absence* of monetary assets in the set of substitution possibilities, while non-gross substitution means their presence.
In each case, only full presence or absence occurs. If one alternative exists, the other cannot. The alternatives are binary opposites, analogous to on/off toggle switches with values, $s = 1$ and $s = 0$.

The logical properties of AA axiom pairs

Four closely related properties flow from AA's conceptualisations. The first is that of *mutual exhaustiveness.* Each axiom pair only allows two possible alternatives, either A or non-A, with no other states being possible. The second is *mutual exclusivity.* If A exists, then non-A cannot exist and *vice versa*. If reality is ergodic, the presence of the required probability distribution eliminates the only other possible state which is the absence of this distribution (non-ergodicity). Similarly, non-ergodicity (the absence of a unique distribution) entirely excludes the only alternative of ergodicity (the presence of this distribution). If money neutrality obtains, real variables cannot have money variables as determinants in the long run, while the inter-mingled causality of non-money neutrality means they can. Finally, gross substitution either holds as a universal proposition across the economy or it does not; resting places or idle balances either exist (non-gross substitution), or do not (gross substitution). An additional reason underpins mutual exclusivity alongside the definitional requirement that each alternative entails the non-existence of the other. This is an axiom's connection to universal truth. Universal truths are always and everywhere true, so that if they exist (or are believed to exist), their negations cannot also exist (or be believed to exist).

The third property is the impossibility of continuous variation of the axioms from one state to the other. There is no possibility of one state slowly morphing into the other where the full

presence of something (eg the probability distribution) gradually fades into its total absence (and *vice versa).* The toggle switch property (s is either 0 or 1) excludes the notion of 'degrees of states' where $0 \le s \le 1$.[12]

The culminating property is that the two possibilities in each pair are *contradictories* in the sense of deductive logic. Contradictories exist when the truth of A implies the falsity of not-A, and the truth of not-A implies the falsity of A. In an AA world with only two alternatives, *non-A is necessarily equivalent to not-A.* Suppose A = reality always has property P, and B = reality never has property P, where P is a property specified by an orthodox axiom. Clearly A and B are not the same, so that B = non-A. But A and B also satisfy the definition of contradictories (the truth of either implies the falsity of the other), so that we also have B = not-A. That is, the AA framework *requires* its non-As to be the contradictories of its As.

Consider the following statements of the three axiom pairs, with the orthodox As followed by their corresponding non-As.
A_E = economies have single unique probability distributions.
A_{NE} = economies do not have single unique probability distributions.
A_{MN} = there is a long run money-real dichotomy.
A_{NMN} = there is no long run money-real dichotomy.
A_{GS} = substitution is always between reproducible commodities.
A_{NGS} = substitution is not always between reproducible commodities.
Applying the contradiction test to each pair, it will be found that A_{NE} = not-A_E and *vice versa*, A_{NMN} = not-A_{MN} and *vice versa*, and A_{NGS} = not-A_{GS} and *vice versa*. That is, all the non-As attributed to the *GT* are the not-As of the As attributed to orthodoxy. In set theory terms, the opposed axiom sets, in pairs and as a whole, constitute disjoint sets with no intersection (see Figure 1 below).

Rejection and choice

General choice situations have multiple alternatives ($n > 2$), so that the rejection of one alternative, say the orthodox axiom set, A_1, leaves a multitude of replacements ($n - 1 \ge 2$). All these are necessarily *different* from A_1, and hence non-A_1s. These non-A_1s vary from those vastly different from A_1 to variously modified versions of A_1, with one unique alternativ being not-A_1 (the full contradictory of A_1). Since AA imposes equivalence on non-A and not-A, it imposes the very special (and extremely restrictive) case of $n = 2$ on the choice set. In AA, if Keynes rejects A_1 and chooses non-A_1, his *only* option is not-A_1.

This AA portrayal is far removed from the *GT.* In rejecting certain key orthodox assumptions, Keynes chose replacements which were *different* from, but not necessarily contradictory to, these assumptions. Indeed, whatever all the orthodox assumptions are (and Keynes sometimes found it difficult to uncover these precisely), and whatever all the assumptions of the *GT* are (his readers also find it difficult to specify these precisely), all the latter are not the contradictories of all the former (since sufficient assumptions can be identified in each theory for this to be established). In replacing old assumptions with new ones, there is a crucial difference between replacement with *different* assumptions and replacement with *contradictory* ones. Rejecting one set opens the door to a vast range of alternatives and does not oblige the adoption of a fully contradictory set.

Thus the first conceptual steps of AA wedge it into a tight logical corner. As explained below, the resulting internal inconsistencies prove fatal to its aim of providing foundations for the *GT* and Post Keynesianism.

AA's theory of general theories and special cases

For AA, it is crucial that the *GT* and Post Keynesianism embrace orthodoxy as a special case.[13] To provide logical foundations for this property, a specific, axiomatic theory of the general theory/special case relationship is advanced.

> A theory is more general if it requires a smaller common axiomatic base than other alternative theories. The latter are special cases that impose *additional restrictive axioms* to the minimum axioms needed for the general theory. (These special cases still rely on the same minimum axioms that underlie the general theory.) (Davidson 1996a p.52)

In other words, Keynes argued that what made his analytical system more general than the classical…analysis is that [his] general theory requires a smaller common axiomatic base (fewer restrictive axioms) than any other alternative theory. Alternative theories then are special cases that impose additional restrictive axioms to the common axiomatic foundation of the general theory. The onus is therefore on those who add the restrictive axioms to the general theory to justify these additional axioms. (Davidson 2006a p.186)

…Keynes argued that what made his analytical system more general than the classical (or Walrasian general equilibrium) analysis is that his general theory requires a smaller axiomatic base (fewer restrictive axioms) than Walrasian theory. Alternative theories such as the Walrasian approach then are special cases that impose additional restrictive axioms to the common smaller axiomatic foundation of Keynes's general theory. (Davidson 2015b p.49)[14]

This theory reposes on three propositions: (a) a general theory has fewer restrictive axioms than its special cases, (b) extra restrictive axioms are added to the axiom set of the general theory to generate these special cases, and (c) the special cases are *theories*. Thus the relationship is theory/axiom-based; the distinction between the two types of *theory* centres on the *number of restrictive axioms*; and the axioms of the general theory are a subset of those of the special theory. Applying this to the Keynes-orthodoxy relationship, orthodox theory is generated by expanding the *GT*'s axiom set by adding extra restrictive axioms.

Six things deserve noting. First, it is a requirement (although never expressly stated) that the additional axioms be logically consistent with the original set. Second, AA advances this theory in completely abstract terms, never examining its implications, explaining actual examples from any field, or providing any historical discussion. Third, this theory does not have origins external to AA, such as in the philosophy of science. It is an internally-generated, self-serving assertion resting on grossly inadequate foundations outlined subsequently. Fourth, the absence of a definition of/criteria for restrictiveness makes classifying and counting relevant axioms mysterious, even though a *sine qua non* of the theory. Restrictiveness is not the simple or self-evident concept it is taken to be. Fifth, no attempt is made to reconcile restrictiveness with universal truth. Finally, the theory is copied directly from orthodoxy for it is identical to the Neoclassical Synthesis/New Keynesian attempt to capture the GT. To orthodox theory as the general theory based on universally flexible prices and zero imperfections, restrictive assumptions are added that introduce rigidities or frictions in the short run, this addition allowing the development of special case 'Keynesian' theories permitting non-money neutrality, lower output and higher (non-involuntary) unemployment in this run.

Keynes's conception of the general theory/special case relationship

The *GT*'s conception of this relationship is utterly different from that of AA. It derives from Keynes's central goal (repeated many times) of developing *a theory of output as a whole*, a theory seen as long neglected to the detriment of economics. From this perspective, his theory is general because of its conclusion that long run equilibrium real output (Y) is a variable capable of taking any magnitude over the range, $0 < Y \leq Y^F$, as distinct from orthodoxy which concludes that $Y = Y^F$ only. In this sense, the orthodox conclusion naturally arises as a special case within his general theory.[15]

That Keynes's conception is incompatible with that of AA is immediately obvious. First, his conception is *conclusion-based, not theory/axiom-based*. The 'case' Keynes singles out is a *magnitude within a conclusion*, not a theory; *a fortiori*, it is not a theory with axioms/assumptions overlapping those of the *GT*.[16] Second, additional assumptions/axioms, restrictive or otherwise, are completely unnecessary to demonstrate that his theory embraces this special case. It already exists as one of the possible conclusions of the original, *unaugmented* assumption set, unaugmented in the sense that no further fundamental assumptions/axioms need to be added for this special case to be *present* as a possibility (although extra restrictions of a quite different kind are, as discussed below, needed for it to be *attained*). Third, Keynes nowhere says that extra assumptions are required for the purpose of demonstrating that orthodoxy is a special case of the *GT*.

That the above conception is textually consistent with the *GT* (and the AA conception textually inconsistent) is also evident from multiple passages, including the notable first

chapter of the GT. Keynes expressly states (my emphases) that the reason he calls his theory general is 'to contrast the *character* of my arguments and *conclusions* with *those* of [classical theory]', which is quite different from saying that it is because classical *theory* is a special case of his general theory. Further, 'the postulates of the classical theory are applicable to a special case only and not to the general case, the situation which it assumes being a *limiting point of the possible positions of equilibrium*'. That is, classical assumptions and classical theory lead to the special case of Y^F, which is but the upper limit of all possible output values in his general theory. In reply to Dimand (2009 p.62) who advanced the variable output proposition, Davidson (2009a p.78-9) revealingly claimed that it was incompatible with the *GT* where 'Keynes argued that there was *not* a whole range of possible equilibria'. Although Davidson's supporting argument rests on semantic confusion and an *ignoratio elenchi*, the important points here are his rejection of this proposition and its inconsistency with the *GT*.[17]

At this point it is instructive to introduce the idea of propositions as *contraries*, rather than contradictories. Propositions are contraries when they cannot both be true. Consider a variation of the previous example that brings it into alignment with the *GT*. A = reality always has property P, and B = reality might have property P. Note that B now includes rather than excludes P, so that the set of all B's possibilities and the set of all A's possibilities overlap with a positive intersection of P_F. The context has also changed, for P now concerns conclusions as in the *GT* (specifically Y^F in the long run), not axioms as in AA. Again we have B = non-A. But since A and B cannot both be true, they are also contraries. They fail the contradiction test because of the positive intersection. Thus the relationship between A and non-A is now very different. In AA, non-A is necessarily contradictory with, and non-inclusive of, A, but in the *GT* the relationship is a contrary one in which a particular feature of A, namely P, is included in non-A as a possibility. The contraries that AA actually needs are conflated with the contradictories it adopts.

Two contexts for assumptions and restrictiveness

Further exploration allows clearer understanding of these inner conflations. Two contexts in which assumptions and restrictiveness are linked need to be distinguished. The first is the standard one where assumptions/axioms are the *initial* foundational propositions deployed in constructing theories and deriving conclusions. These assumptions then restrict the conclusions and applicability of the theory. The second is that of taking an existing theory and adding something extra so as to constrain the theory into delivering a specified conclusion. AA conceptualises this extra thing as an *additional* set of restrictive assumptions/axioms. In the former context, the fundamental assumptions/axioms are used to *derive the theory*, with no restrictions placed on its conclusions beyond logical entailment. In the latter, the theory is *already* derived and the extra addition restricts/obliges the theory to *generate the prescribed conclusion*. To distinguish these different contexts, they are here called the 'general theory derivation context' and the 'special case generation context'.

AA conflates the two contexts, presuming that both have the same nature. It begins with a general definition of assumptions/axioms, viewing them as propositions 'underlying' a theory and as 'the basis for developing a theory'. That is, it begins with assumptions/axioms taken in their general theory derivation sense. Then, in advancing its theory of general theory-special case relationships, it takes the extra restrictive assumptions/axioms it requires to be *the same type* as those used in the original theory derivation situation, and so is led to the idea that the special case is a theory with the same axiomatic form as the parent general theory. That this is mistaken can be clarified with two examples.

The theory of friction-based motion begins with basic assumptions/axioms, one of which postulates a variable friction coefficient, to derive a general theory of the subject. Then, to understand frictionless motion, the coefficient is set to zero to obtain the special case which may be described as the theory of frictionless motion. Two features are important here. The value of a variable within the *original* assumptions is restricted to a particular value, and what results is a *theory* that is derived from, and entirely consistent in all other respects, with the general theory. From the viewpoint of the general theory, it already embraces the theory of frictionless motion, the latter simply being a special case of itself based on a specific value of a variable in an existing assumption. But from the viewpoint of the special case, it may be viewed as a distinct *theory*, even if derived from an identical set of fundamental assumptions apart from one where what was previously variable is now fixed. As outlined previously, this is entirely analogous to orthodoxy's attempt to capture the *GT* as its special case.

Now consider the quite different situation of the *GT*. The special case is a particular *conclusion* of an *alternative* theory of the subject; it is *not* that alternative theory itself. The special case is the magnitude of Y that orthodoxy concludes will happen in the long run (Y^F), not the orthodox theory (in part or in whole) that led to this conclusion. While one can derive a *special case theory* of frictionless motion from the general theory of friction-based motion because they have identical assumption sets apart from the restriction on the coefficient, it is impossible to derive orthodox *theory* from the *GT* because they have very *different* assumption sets.

Nor, in arriving at the orthodox conclusion within the *GT* is it simply a matter of altering the value of one or more parameters. What is required within Keynes's general theory is not the addition of more restrictive assumptions/axioms in the AA sense, but the imposition of restrictions on the behaviour of the key determinants in that theory so that they arrive at the stipulated special conclusion. Suppose we start with low Y and enquire as to what conditions will be necessary within the *GT* to reach Y^F. The answer will take the form of a complicated set of restrictions on its internal variables. For example, we will need suitable trajectories (a) for the state of confidence, say that it starts with sufficient optimism to initiate growth, which optimism then moderates in exactly the right way for the economy to reach the full employment (and constant inflation) destination, and (b) for all other variables so that they behave in ways that never prevent this outcome. We are thus imposing restrictions on the *behaviour of variables within the existing set of assumptions/axioms*, not adding extra fundamental axioms to this set.[18]

One should be wary here of being misled by ordinary language. In conversation, one might describe the above situation as one of making extra restrictive 'assumptions' and then adding them to Keynes's theory. However, this is just *a manner of speaking* with *no* analytical import. For these kinds of 'assumptions' are *analytically equivalent* to the imposition of requirements on existing variables, which indicates that 'assumption' in this usage does not have the same meaning as it has in describing the fundamental assumptions from which theories are derived. This notion of assumption belongs to the internal special case generation context, not the general theory derivation context. Less importantly, the non-equivalence of the two senses also follows from the AA definition of axioms. There is an understandable (if mistaken) sense in viewing the axioms/assumptions in the general theory derivation context as universal truths, but it is nonsense to view the different kind of axioms/assumptions used in the special case generation context as universal truths. Neither the restrictive assumption of zero friction, nor the complicated additional restrictions needed for the *GT*, can possibly be regarded as universal verities.

These matters also relate to specifying the conditions of applicability of an existing theory. To say 'a theory holds on given assumptions' is an alternative way of saying that the applicability of the theory is restricted to situations described by the assumptions. But because this manner of speaking can be used in *both* contexts outlined above (it does not distinguish whether 'assumptions' are fundamental or merely subsequent restrictions on the behaviour of existing variables), it can generate the misconception that 'assumption' has the same sense in both contexts. In short, AA's error is twofold; it reconceptualises special cases as the *theories* that originally gave rise to particular conclusions rather than the particular conclusions themselves, and it confuses that which is external and necessary in the derivation of a theory with what is subsequent and internal to the general theory in making it deliver special outcomes.

The issue of applicability is also key to understanding the internal origins of AA's general theory-special case theory. It derives from Davidson's questionable translation and interpretation of two of the three sentences missing from the English translation of Keynes's German preface to the *GT*. Schefold provides an accurate translation as follows.

> This is one of the reasons which justify calling my theory a *general* theory. Since it is based on less narrow assumptions than the orthodox theory, it is also more easily adapted to a large area of different circumstances. (Schefold 1980 p.175)[19]

Davidson, however, in a very doubtful, far less defensible translation, substitutes 'fewer' for 'less' (and also axiom for assumptions),[20] which significantly changes the meaning of the sentence. Then, by ignoring the context in which the remarks are made, he arrives at his AA

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interpretation: the *GT* is more general because it has fewer (a smaller number of) restrictive axioms, and since orthodox theory is its special case, orthodoxy must have a greater number of such assumptions. Hence adding these extra restrictive assumptions to those of the *GT* allows orthodox theory to be derived from the *GT*.

This AA construction is beset by further difficulties, however. First, Keynes's authorship of this sentence is far from certain. Second, the more accurate translation from the German is 'less narrow assumptions' which is critical because 'fewer' refers to *number* while 'less' refers to *nature*. Adopting fewer axioms as the correct translation immediately generates the ideas of counting axioms and of larger and smaller axiom sets. By contrast, 'less' eliminates any necessity of axiom counts and focuses attention on the broader idea of *different* axiom sets, a variety of which could be less restrictive quite independently of numbers of axioms.

Third, Keynes's remarks, before, within and after these sentences, provide the relevant but ignored context that what he is commenting on here is the *applicability* of the *GT*, not axiom counts.

> …I confess that much of the following book is…expounded mainly with reference to the Anglo-Saxon countries. Nevertheless the theory of output as a whole, which is what [it] purports to provide, is much more easily adapted to the conditions of a totalitarian state, than is the [orthodox] theory. …[Here the first two missing sentences appear, as quoted above, followed by the third missing sentence as follows]. Although I have thus worked it out having the conditions of the Anglo-Saxon countries in view – where a great deal of *laissez-faire* still prevails − it yet remains applicable to situations in which national leadership is more pronounced. (Schefold 1980 p.175)

From an argument saying that the *GT* has greater applicability due to having *less* restrictive assumptions, no conclusion follows that orthodox theory is deducible from the *GT* by adding extra (restrictive) assumptions. This AA conclusion is a *non-sequitur*. The two assumption sets are simply *different*, those of the *GT* having greater applicability because they are *less* restrictive. AA's mistake is to conflate less with fewer, that is, to confuse a change in kind with a change in number.[21]

Geometry **and AA**

Exemplars from other fields might possibly support the AA theory of general theory-special case relationships, but the significant case of Euclidean and Non-Euclidean geometry does not. Keynes referred to these geometries in Chapter 2 of the *GT* at some length, stating that just as Non-Euclidean geometry needed to throw over the 'axiom of parallels' of Euclidean geometry, so economics needed to do 'something similar' by throwing over certain postulates of classical theory to develop a general theory. Davidson's writings constantly refer to these remarks in a metaphorical manner implying an identity or *exact correspondence* between the two upheavals.

> Keynes developed the economic analogue to non-Euclidean geometry …More than half a century later, Keynes's 'non-Euclidean' revolutionary approach remains undiscovered …(Davidson 2011 pp.5, 8).

> Keynes compared those economists whose theoretical logic was grounded on the classical special case [of] additional restrictive axioms to Euclidean geometers… In light of Keynes's analogy to geometry, Post Keynesian…theory might be called non-Euclidean economics. …Just as in non-Euclidean geometry…, [so] in the Keynes-Post Keynesian non-Euclidean economic world,… (Davidson 2006a pp.187, 189).[22]

If Keynes's theory is non-Euclidean and if it embraces the overthrown orthodox theory as a special case, the implication is that the overthrown Euclidean geometry is a special case of non-Euclidean geometry. Applying AA theory, both special cases are derived from the respective general theories using additional restrictive assumptions. Just as in economics, so in geometry; the overthrown Euclideanism is then a special case of Non-Euclideanism.

Unreflective conclusions of this kind are false and misleading. Essential to Euclidean geometry is the axiom of parallels, the proposition that, given one straight line and a point P not on this line, *only one* parallel line can be drawn through P. Dissatisfaction with this

axiom for centuries led to the development of Non-Euclidean geometries, the family name for all geometries based on axioms *negating* Euclid's axiom of parallels. Prominent examples are Riemannian geometry based on the axiom that there are *no* lines through P parallel to the initial line, and hyperbolic geometry based on the axiom that *two or more* parallel lines can be drawn through P. Both these Non-Euclidean geometries have the property that Euclidean geometry cannot be logically derived as their special case.   They are entirely different theories with incompatible assumption sets. From axioms negating Euclid's axiom (no lines, or multiple lines), we cannot reach conclusions derived from his axiom (only one line), even if further (logically consistent) axioms were to be added. Hence Euclidean geometry is not a special case of Non-Euclidean geometry, and exact correspondence or 'parallelism' breaks down. Again it is a situation of mutual exclusivity in the two axiom sets, rather than inclusiveness.

The more accurate reading is that Keynes's remarks constitute a *simile* expressing the idea that just as there was a *revolutionary change* in geometry, so there needs to be a *similar* change in economic theory. And while this naturally involves rejecting earlier assumptions and replacing them with new ones, there is no implication that these changes actually possess the same properties. Keynes's use of a simile rather than anything resembling a metaphor allows for asymmetries between the geometrical and economic fields.[23]

First internal inconsistency

AA cannot derive orthodox theory as a special case of the *GT*. Simple diagrams assist in demonstrating this. Let A and C refer to sets of assumptions/axioms and conclusions, and the subscripts C and K to classical theory and Keynes's theory, respectively. A superscripted dash distinguishes the *GT*'s actual versions of A and C from their AA counterparts.

INSERT FIGURE 1 ABOUT HERE

Figure 1 shows the initial axiomatic structure advanced by AA concerning the fundamental differences between the *GT* and orthodoxy. Since A_K and A_C are contradictories, their conclusions, C_K and C_C, are also contradictories. The impossibility of intersection of the axioms results in the impossibility of intersection of the conclusions.   It is then impossible for orthodoxy to be a special case of the *GT*, either in the AA sense of the *GT* embracing the whole of orthodox theory, or in the sense of the *GT* embracing the orthodox conclusion that long run output always equals Y^F.

Can AA's theory of general theory-special case relationships come to the rescue? Consider Figure 2. To Keynes's *general theory* in its AA form (the rectangle containing A_K and C_K), extra restrictive axioms, A_A, are added to generate the special case of *orthodox theory* (the rectangle with $A_K + A_A$ and C_C). But this merely displaces, without solving, the logical difficulties. Since $A_K \equiv$ not-A_C, C_C has to be derived from an axiom set containing the contradictories of the axioms previously necessary to deriving C_C. What sense, then, is to be made of $A_K + A_A$? It is clearly very different from A_C, the axioms of ergodicity, money neutrality and gross substitution, for it comprises a combination of (i) A_K, the axioms of non-ergodicity, non-money neutrality and non-gross substitution, all being the *contradictories* of their A_C counterparts, and (ii) A_A, an as yet unspecified set of axioms. Since $A_K + A_A \equiv$ not-$A_C + A_A$, is it possible to derive orthodox theory from an axiom set which includes axioms contradictory to those necessary to the derivation of that theory?

INSERT FIGURE 2 ABOUT HERE

If this question has an answer, it lies with A_A. What, then, are A_A's constituents? These are never discussed and so remain mysterious. What, then, *might* they be? They have to perform a daunting task. We are told that orthodox theory is grounded on A_C, and also that it can be derived from not-$A_C + A_A$. To achieve the latter, A_A has the fivefold task of (a) being *consistent* with A_K or not-A_C, (b) *preventing* not-A_C from generating conclusions contradictory to those arising from A_C, (c) obliging A_K to deliver the *same* set of conclusions as arise from A_C, (d) remaining *restrictive* in some unspecified sense, and (e) retaining a connection to *universal truth*. Using the ergodicity/non-egodicity pair to illustrate, A_A must be consistent with non-ergodicity, prevent non-ergodicity from delivering conclusions contradictory to those derived from ergodicity, make non-ergodicity deliver the same

conclusions as ergodicity, while also being restrictive in an undefined way and maintaining links to universal truth.

My claims are that this task is simply impossible, the required additional assumptions do not exist, and any assertion to the contrary is merely magic wand waving.[24] Intuitions of the absurdity of deriving Walrasian theory from the assumptions of the *GT* by adding some extra assumptions are entirely justified. To avoid charges of camouflaged nonsense or logical alchemy, AA advocates need to demonstrate that such claims are mistaken, and show how a theory built on given assumptions can logically emerge, even when aided by further assumptions, as a special case of a theory built on the contradictories of these given assumptions.

INSERT FIGURE 3 ABOUT HERE

Now consider matters from the viewpoint of the *GT*. Figure 3 illustrates its straightforward conclusion-based general theory-special case relationship. From its actual assumptions, A_K', are derived its conclusions, C_K', a central one of which is the output conclusion $0 < Y \leq Y^F$ which clearly contains Y^F as a possibility. This is the sense in which Keynes claims the *GT* has orthodoxy as a special case.

INSERT FIGURE 4 ABOUT HERE

Figure 4 then shows the situation required to turn this possibility into reality. Some behavioural restrictions, R, are superimposed on A_K' such that the orthodox value of Y is generated within C_K'. The arrow that led from A_K to C_K in Figure 1 is now obliged to lead only to the output level common to C_K' and C_C, namely Y^F. No additional (fundamental) assumptions/axioms are involved; all we have are restrictions imposed on the behaviour of existing variables.

In fulfilling this necessary and self-appointed task, AA fails completely. Its logical architecture prevents it from synthesising its axiom sets and its general theory-special case theory, so that its claims here join the list of 'impossibility theorems' engendered by much AA reasoning. To claim otherwise is to practise the 'charlatanry' which Keynes constantly criticised.

Second internal inconsistency

Two conflicting propositions about *uncertainty* are advanced simultaneously. One is that the *GT* is based on radical uncertainty and that the only ontological state consistent with this is non-ergodicity. The other is that agents know with certainty in the pre-infinity here and now that the world they inhabit is non-ergodic. But since human agents cannot possibly know this (see below), AA surreptitiously introduces an assumption of *perfect knowledge* of the prevailing ontological state. A theory which declares that agents exist in an environment of irreducible uncertainty is actually dependent on an assumption that agents have perfect fore-knowledge that their ontological state is non-ergodic. Those who are portrayed as ignorant of what the future holds somehow have the information that data observed over infinite time and space prove the existence of non-ergodicity.[25]

Two important consequences follow from agents' inabilities to identify ergodicity and non-ergodicity. One is that the ontological approach to uncertainty becomes prisoner to the epistemological approach since abilities/inabilities to know are central to the latter. The other is that, from the agent's viewpoint (the one that matters), irreducible uncertainty is omnipresent in *ergodic* realities; if reality actually is ergodic, agents can never know this and its associated probability distribution. And if probabilities are unavailable, irreducible uncertainty exists.

Third internal inconsistency

This arises from the combination of the AA definition of an axiom and its two opposed axiom sets. If axioms are taken in the strong sense of being universal truths, then they are always and everywhere true. But the axiom pairs within each theory are contradictory, which means the opposed propositions in each pair cannot both be true. If A is universally true, it cannot be the case that non-A (or not-A) is also universally true. At all times and places,

characteristics cannot be both present and absent, and propositions both true and false.

If axioms are taken in the weaker form of only being *believed* to be universal truths, related problems arise. First, many orthodox economists do not actually believe in the universal truth of their assumptions. The four leading mainstream economists regularly cited by Davidson exemplify this. Samuelson (1968 pp. 5, 12) accepts that his assumptions are heroic, oversimplified abstractions and that modern economists make 'economics out of fairy tales'; Lucas (1981 p.271) declares that new Classical economics is 'patently unreal'; Blanchard (1990 p.828) thinks long run money neutrality is based more on faith than empirical evidence; and Friedman's (1953 ch.1) 'as if' methodology makes a virtue out of employing false assumptions so long accurate predictions emerge.[26] Second, as indicated above, Keynes never advanced his assumptions as universal truths. Third, the impossibility of determining whether reality is ergodic or non-ergodic means that no-one, including Post Keynesians, can scientifically believe that claims asserting the existence of either state are universal truths. Finally, how can one believe that from a theory based on one set of universal truths a special case of that theory can be derived, when the latter requires a contradictory set of universal truths? One set of universal truths cannot exist (or be believed to exist) alongside an opposed set.

Fourth internal inconsistency

This is a potential, rather than actual, inconsistency, one awaiting any AA advocate seeking to avoid the first inconsistency by taking refuge in non-ergodicity. The *scientific* definition of ergodicity used by Davidson is as follows.

> *Time statistics* refer to statistics calculated from a single realization over any period of calendar time. *Space statistics* are statistics formed over a universe of realizations at a fixed point of time…. If the stochastic process is *ergodic*, then for infinite realizations, the space and time statistics will coincide. For finite realizations, the calculated space and time statistics may differ…, but they will tend to converge…as the number of observations increase. (Davidson, 1996b p.480n3; also 1994 p.89-90; 1988 p.331-2)

With ergodicity so defined, non-ergodicity then comprises every possible state without a single probability determinant of all time and space statistics.

It follows that accurate agent identification of an ontology as ergodic involves three conditions – the existence (and collection) of time statistics to infinity, the existence (and collection) of space statistics to infinity, and identity in the probability distributions delivered by the two series. Similarly, identification of an ontology as non-ergodic must satisfy three conditions – the existence /collection of time statistics to infinity, the existence/collection of space statistics to infinity, and the *non-identity* of whatever distributions are delivered by the two series. In either case, it is *only* when infinity 'arrives' in time, and infinity 'embraces' all of space, that human agents can determine whether the ontology they inhabit is ergodic or non-ergodic (whether in reality or in models). The obvious conclusion is that humans can *never* determine the nature of the ontological state in which they conduct their economic activities.[27]

But indubitable knowledge of ontological states is essential because AA renders rational behavior (and hence economic activity, survival and prosperity) ontology-dependent. Under ergodic conditions, rational agents are theorized as using the underlying probability distribution in all decision-making; byaccurately aligning their decisions with the nature of reality, correct forecasting and economic success are achieved over time and space (at least on average). And, under non-ergodic conditions without a unique probability driver, rational agents are theorized as using non-probabilistic decision procedures that essentially amount to 'embrace uncertainty and have a shot'.

An AA advocate might try to dodge the first inconsistency, however, by arguing that in non-ergodic states anything can happen, and hence that full employment can be attained as special case. Without a unique probability distribution and with agents taking unquantifiable chances and trying to create their own futures through enterprise, $Y = Y^F$ is a perfectly possible outcome. Thus non-ergodic worlds can generate a range of output levels *a la* Keynes, within which Y^F is a special case.

This argument involves two internal inconsistencies. First, there is an illicit switch in the conception of the special case. AA's theory-based concept of a special case has been abandoned and replaced by the *GT's* conclusion-based concept. Although more in sympathy with the *GT*, the substitution is internally inconsistent with AA. Mid-argument changes of concepts, however tempting, undermine logical validity and invite *ignoratio elenchi*.

Second, even if AA's theory of special cases is abandoned, another obstacle emerges. The above argument depends on agents (in the model or in reality) knowing, with certainty here and now, that the reality they inhabit is non-ergodic. But, as indicated above, such knowledge is impossible for agents (and theorists) ever to obtain. The consequence is that rational agents in the AA account are frozen into inaction because they do not know how to behave rationally. The economic activity needed to generate output never gets underway, let alone reaching Y^F. It is then inconsistent to claim that full employment could be reached in non-ergodic systems populated by rational agents when it is impossible for *any* level of output to be generated due to the absence of agent knowledge of the ontological state.[28]

To sum up the discussion of logical inconsistencies, no-one, Post Keynesian or otherwise, can advance rational economic thought by holding inconsistent views simultaneously.

AA's reframing of the *Treatise on Probability*

Here AA introduces remarkable conceptual and textual inconsistencies. In relation to the *TP*, it is obliged to develop a reframed version compatible with AA and opposed to rival interpretations. Although its appeals to the *TP* are infrequent, its interpretation has the same binary form displayed in its economic domain. Here Knight is its chief mentor − his risk-uncertainty distinction (risk and uncertainty are mutually exclusive categories), and the relative frequency theory of probability (underpinning his distinction and its related theory of profit) are fully embraced. Their combination generates a *probability-uncertainty divorce* in which two states of knowledge inhabit different universes with nothing in common. These states are (a) the presence of certainty, both absolute and actuarial (or probabilistic), and (b) the presence of uncertainty, that is, the absence of both absolute and actuarial certainty. By excluding *probabilistic uncertainty*, this directly aligns with the property of mutual exclusivity in the AA axiom pairs used for orthodoxy- *GT* differences.

The fact that the *TP's* ideas do not fit this binary structure destroys AA's attempted dovetailing of Keynes's two main works. As is well known (and surely textually incontrovertible), the *TP*, from its first page onwards, advances an indissoluble *probability-uncertainty connection*, and in this context develops the logical theory of probability, which is entirely different from the frequency theory. In the *TP*, the existence of uncertainty first gives birth to probability, and hence to probabilistic uncertainty (on the assumption that humans can discern these probabilities), and then gives birth to other children related to non-probabilistic or irreducible uncertainty, these occurring when probabilities (in one form or another) are unavailable so that humans cannot reduce uncertainty to the required probabilities. As with the *GT*, AA's reframing of the *TP* derives from external, alien elements, and leads to interpretations incompatible with the actual contents of both works.

Diagnosis

Several causes contribute to AA's multiple failures, all being central to its global, axiom-based reframing strategy. The first concerns departure points. AA begins, *not* with concepts to be found in Keynes's own writings, but with a set of external ideas, including, especially, stochastic process theory and Knight's risk/uncertainty separation. This route is taken, not for analytical reasons, but for the external strategic purpose of developing a 'modernised' reinterpretation of Keynes's thought so as to facilitate dialogue with orthodoxy. Next, regarding the *GT*, it does not start with its positive content but with its attributed negative content; that is, with the three orthodox axioms that the *GT* purportedly rejects. Their rejection then delivers the three axioms supposedly underpinning the *GT*, with the movement from negative to positive occurring via contradictories not contraries. If orthodoxy asserts A, then Keynes asserts non-A, which in AA is identical to not-A. Although rejection normally allows many choices, AA only allows that of not-A. Also 'modernised' is the *TP* so that it is compatible with AA themes, especially concerning Knight's treatment of knowledge and uncertainty. Third, this assemblage of ideas is heavily axiomatised in a highly purposeful and scientifically-garbed manner: a bizarre (and 'unmodern') conception of axioms, axiom-based

oppositions between the *GT* and orthodoxy, the axioms of stochastic process theory, an axiom-based theory of general theory-special case relationships, the axioms underlying Knight's risk/uncertainty distinction; and an axiom asserting the relative frequency interpretation of probability. Finally, to give plausibility to its arguments, AA relies on a variety of conflations, including axioms (in its sense) with assumptions, axioms with restrictions on existing variables, theories with conclusions, fewer with less, contradictories with contraries, metaphors with similes, and Knight with Keynes on uncertainty.

What emerges from this AA 'production function' is a thoroughly reframed version of Keynes's thought with two main consequences. The AA version is conceptually and textually inconsistent with his writings, and any and all logical inconsistencies concealed within this externally-generated framework are transferred into AA versions of Keynes and Post Keynesianism. The former introduces serious fictions into the *GT* and *TP*, and the latter renders AA unfit for scientific work. The project also fails, unsurprisingly, in generating greater dialogue with mainstream economists. If the AA framing can be completely removed, value still remains in Davidson's writings, but its dominating presence, despite laudable intentions aimed at reviving Keynes, means that AA joins the list of approaches that distort and side-track the *GT* and Post Keynesianism.

Conclusion

There is no dispute that the assumptions, conclusions and methodology of the *GT* are very different from those of orthodoxy, including non-probabilistic uncertainty, non-money neutrality in all runs, substitutions between monetary and non-monetary items, and full employment as a special case in a spectrum of equilibrium output levels. What has been argued is that, on logical, conceptual and textual grounds, Davidson's axiomatic approach fails dramatically to capture these key features in intellectually respectable ways. To Keynes's pen, AA attributes arguments and ideas that never appear in his writings or are implied by them. As someone philosophically practised in logic and the meanings of concepts, Keynes would never have wittingly granted passage to this ensemble of logical and conceptual inconsistencies. One might recall Davidson's own advice (2007 p.27; 2002 p.41) that 'before accepting the conclusions of any economist's theory as applicable to the world in which we live, the careful student should always examine and be prepared to criticize the applicability of the fundamental axioms of the theory'. The only necessary addition here is critical analysis of the theory's internal logic. If true that 'theorists rarely make logical errors in moving from axioms to conclusions' (Davidson 2009b p.31), then AA is one of the rarities.

For whatever reason, possibly an appearance of being all-encompassing, tightly constructed and scientific-sounding, AA has previously avoided comprehensive scrutiny and its many failures have escaped detection. The cause of its sinking to the depths without hope of salvage, however, is its own design faults. The only alternative for Post Keynesianism is to transfer to a sea-worthy vessel capable of successfully navigating these waters, one with a design that starts with the actual features of the *GT* − its own assumptions, approach to uncertainty, mode of analysis, and relationship to orthodoxy – and proceeds from there in ways that are, *inter alia*, respectful of logic.

References

Blanchard O. (1990) 'Why Does Money Affect Output? A Survey.' In B.M. Friedman and F. H. Hahn (eds.), *Handbook of Monetary Economics,* vol. 2, New York: North Holland.
Davidson P. (1980a) 'The dual-faceted nature of the Keynesian revolution: money and money wages in unemployment and production flow prices'. *Journal of Post Keynesian Economics*, 2 (3), 291-307.
Davidson P. (1980b) 'Post Keynesian economics: solving the crisis in economic theory'. *The Public Interest*, Special Issue, 151-73.
Davidson P. (1982-83) 'Rational expectations: a fallacious foundation for studying crucial decision-making processes, *Journal of Post Keynesian Economics*, **5**(2), 182-198.
Davidson P. (1984) 'Reviving Keynes's revolution'. *Journal of Post Keynesian Economics*, 6 (4), 561-75.
Davidson P (1988) 'A technical definition of uncertainty and the long-run non-neutrality of money'. *Cambridge Journal of Economics*, 12(3), 329-337.
Davidson P. (1989) 'Keynes and money', in Hill R. (ed) *Keynes, Money and Monetarism*,

Basingstoke: Macmillan, 2-26.

Davidson P. (1991) 'Is probability theory relevant for uncertainty? A Post Keynesian perspective', *Journal of Economic Perspectives*, 6(1) 129-43.

Davidson P. (1994) *Post Keynesian Macroeconomic Theory*, Aldershot: Edward Elgar.

Davidson P. (1996a) 'What revolution? The legacy of Keynes'. *Journal of Post Keynesian Economics*, 19(1), 47-60.

Davidson P. (1996b) 'Reality and economic theory'. *Journal of Post Keynesian Economics*, 18(4), 479-508.

Davidson P. (1996) 'Some misunderstanding on uncertainty in modern classical economics'. In Schmidt C. (ed) *Uncertainty in Economic Thought*. Cheltenham: Edward Elgar. 1996. pp.21-37.

Davidson P. (2000) 'There are major differences between Kalecki's theory of employment and Keynes's general theory of employment interest and money. *Journal of Post Keynesian Economics*, 23(1), 3-25.

Davidson P. (2002) *Financial Markets, Money and the Real World*, Cheltenham: Edward Elgar.

Davidson P. (2003-04) 'Setting the record straight on a history of Post Keynesian economics'. *Journal of Post Keynesian Economics*, 26(2), 245-72.

Davidson P. (2005) 'The Post Keynesian school', in Snowdon B, and Vane H. (2005) (eds) *Modern Macroeconomics*, Cheltenham: Edward Elgar.

Davidson P. (2006a) 'Samuelson and the Keynes/Post Keynesian Revolution'. In Szenberg M., Ramrattan L. and Gottesman A. (eds) *Samuelsonian Economics and the Twenty-First Century*, Oxford: Oxford University Press.

Davidson P. (2006b) 'Keynes and Money'. In Arestis P, and Sawyer M. (eds) *A Handbook of Alternative Monetary Economics*: Cheltenham: Edward Elgar.

Davidson P. (2007) *John Maynard Keynes*, Basingstoke: Palgrave Macmillan.

Davidson P (2009a) 'Reply to contributors to the discussion of the central themes of *John Maynard Keynes*'. *Journal of Post Keynesian Economics*, 32(1), 73-81.

Davidson P. (2009b) *The Keynes Solution*, New York: Palgrave Macmillan.

Davidson P. (2009c) 'Risk and uncertainty in economics', conference paper, Westminister, London. Available at http://econ.bus.utk.edu/documents/davidsonpapers/ RISKANDUNCERTAINTYINECONOMICS.pdf

Davidson P. (2011) *Post Keynesian Macroeconomic Theory*, 2nd ed., Cheltenham: Edward Elgar.

Davidson P (2015a) 'A rejoinder to O'Donnell's critique of the ergodic/nonergodic explanation of Keynes's concept of uncertainty'. *Journal of Post Keynesian Economics*, 38, 1-18.

Davidson P. (2015b) *Post Keynesian Theory and Policy*, Cheltenham: Edward Elgar.

Dimand R.W. (2009) 'Central themes of Paul Davidson's *John Maynard Keynes*', *Journal of Post Keynesian Economics*, 32 (1), 159-72.

Ehnts D. and Alvarez M. (2014) 'Samuelson and Davidson on ergodicity', paper to the 12th International Post Keynesian Conference, University of Missouri-Kansas City.

Friedman M. (1953) *Essays in Positive Economics*, Chicago: Chicago University Press.

Hagemann H. (2014) 'The German edition of Keynes's General Theory: controversies on the preface', *Research in the History of Economic Thought and Methodology*, 32, 159-66.

Hodgson G. (2001) *How Economics Forgot History*, London: Routledge.

Keynes, J.M. (1921), *Treatise on Probability*, Volume VII in *The Collected Writings of John Maynard Keynes*, London: Macmillan.

Keynes, J.M. (1936), *The General Theory of Employment, Interest and Money*, Volume VII in *The Collected Writings of John Maynard Keynes*, London: Macmillan.

Keynes, J.M. (1939) 'Relative movements of real wages and output', *Economic Journal*, 49(193), 34-51.

Lucas R. (1981) *Studies in Business Cycle Theory*, Oxford: Blackwell.

O'Donnell, R.M. (2013) 'Two Post-Keynesian Approaches to Uncertainty and Irreducible Uncertainty'. In *Oxford Handbook of Post-Keynesian Economics*, Harcourt G.C. and Kriesler P. (eds), vol. 2, Oxford University Press, 2013, 124-42.

O'Donnell, R.M. (2014) 'A critique of the ergodic/nonergodic approach to uncertainty'. *Journal of Post Keynesian Economics,* 37(2), 187-209.

O'Donnell R.M. (2016a) 'Second Contribution to the ENE Critique: Reply to Davidson, Part 1', *Journal of Post Keynesian Economics*, 39, XXXX.

O'Donnell R.M. (2016b) 'Third Contribution to the ENE Critique: Reply to Davidson, Part 2', *Journal of Post Keynesian Economics*, XXXX.

Samuelson P. A. (1968) 'What Classical and Neoclassical monetary theory really was',

Canadian Journal of Economics, vol 1, no. 1, 1-15.

Schefold B. (1980) 'The General Theory for a totalitarian state? A note on Keynes's preface to the German edition of 1936', *Cambridge Journal of Economics*, 4, 175-176.
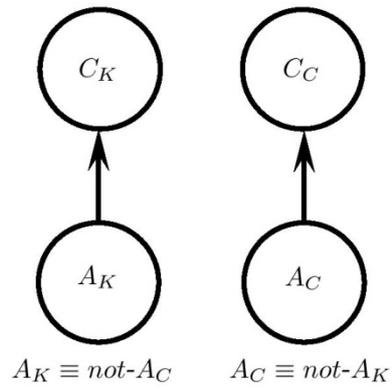
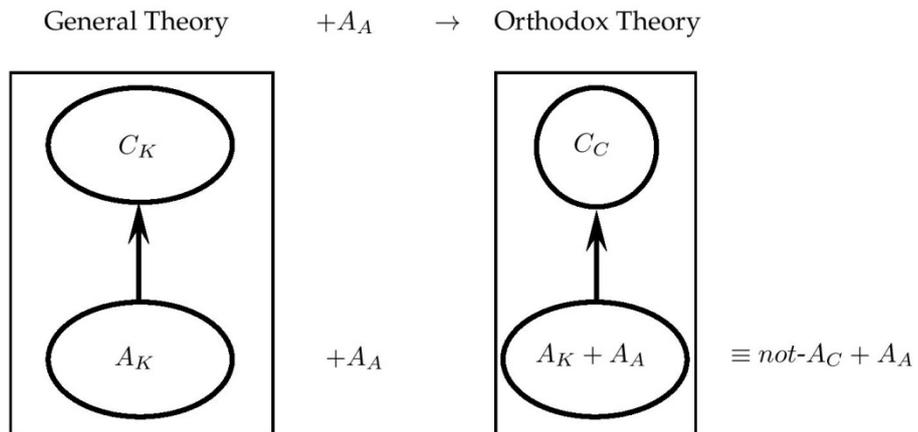Figure 1: AA's axiomatic portrayals of the GT and orthodoxy

Figure 2: AA's theory of general theory-special case relationships applied to the GT
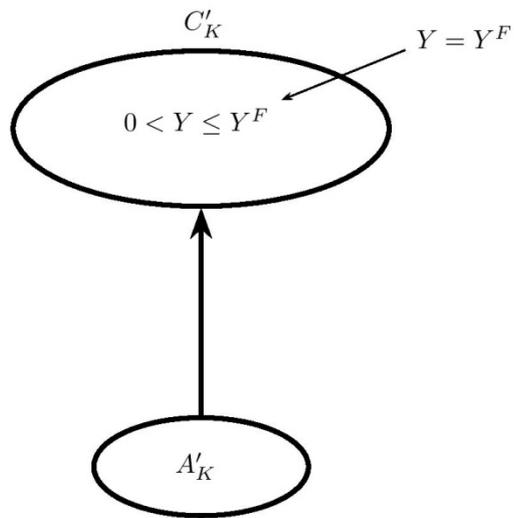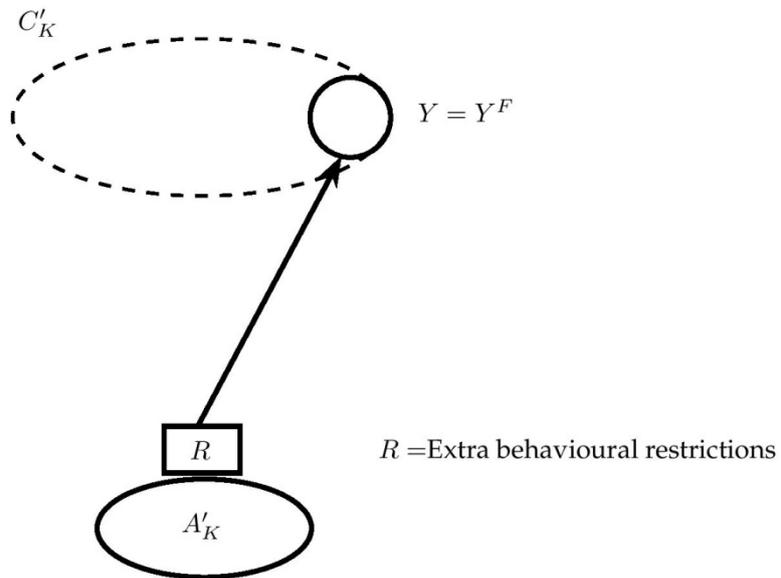
Figure 3: The GT's actual logical structure

Figure 4: Extra restrictions on existing GT assumptions to generate the orthodox long run output level as a special case

Endnotes

[1] For example, Dimand (2009) questions the necessity of the gross substitution axiom to orthodoxy, O'Donnell (2013, 2014, 2016a, 2016b) criticises Davidson's ergodic/nonergodic distinction, and Ehnts and Alvarez (2014) challenge Davidson on Samuelson.

[2] The (discussible) adequacy of AA's representation of orthodoxy is largely bypassed here.

[3] *No* claim is made that these are the only logical, conceptual or textual flaws.

[4] All quotations in this paragraph are from the *GT*, ch 8. See also Keynes (1939 p.40) where an argument supporting an inverse short period output-real wage relationship is characterised as 'subject, of course, to various qualifications in particular cases, but remaining a reliable generalisation by and large'.

[5] See O'Donnell (2016a pp.XXX).

[6] See also Davidson (2011 pp.9, 17; 2007 pp.26-34; 1996 p.494n18; 1994 p.17; 1984 p.562). The three classical assumptions Keynes rejected at the end of chapter 2 of the *GT* are quite different from the AA axioms and difficult to align with them individually and collectively.

[7] See, for example, Davidson (1982-83, 1988 p.332-6, 1989 pp.6, 24n15, 1991 p.142, 2002 pp.54, 59 and 2007 pp.34, 87, 113-4, 204).

[8] This requirement is impossible to satisfy; see O'Donnell (2014, 2016a, 2016b). For further discussion of the ergodic/non-ergodic distinction, see Davidson (2011 pp34-9, 2002 pp.46-59, 1996b, 1996c, 1988, 1982-83) and O'Donnell (2014, 2016a, 2016b).

[9] For further discussion of the money neutrality/non-money neutrality distinction, Davidson (2006b, 2002 pp.41-2, 1989, 1988, 1984).

[10] Being a unit of account is a necessary but insufficient condition of being money.

[11] For further discussion of the gross substitution/non-gross substitution distinction, see Davidson (2011 p.56-60, 2007 pp.82-87, 2006b, 2002 pp.152-7, 2000, 1984, 1980a, 1980b).

[12] Note that *within* non-gross substitution, there may be variable amounts of substitution between reproducibles but, since this state is defined as the absence of *universal* gross substitution, there cannot be degrees of the state itself.

[13] See, for example, Davidson (2016 p.1).

[14] See also Davidson (2015a pp.7-8, 2011 p.36, 2007 p.170, 1996b p.494).

[15] For completeness, the condition of steady inflation may be added in both cases.

[16] Strictly speaking, the conclusion within orthodoxy is that Y^F is unique, while the conclusion in the *GT* is that it is not unique, just a possible magnitude.

[17] It is also inconsistent with other statements by Davidson that equilibria in the *GT* are not confined to Y^F.

[18] Note in passing that exactly the same approach applies to each of the infinite number of possible outcomes in Keynes's analysis, none of which, except one, have significance as a special case. Consider what highly contrived requirements on variable behaviour would be required to generate $0.7143Y^F$ in the long run.

[19] Hagemann (2014 p.163) also accepts this translation.

[20] See, for example, Davidson (2015a p.7, 2015b p.49, 1996a p.52, 1996b p.494).

[21] For further discussion, see O'Donnell (2016a). The *GT*'s assumption set is less restrictive than those of *both* orthodoxy and AA's version of the *GT*.

[22] See also Davidson (2007 p.35, 2006b pp.140, 152&n6, 2005 p.454, 1996a p.52).

[23] See also his discussion of analogy and likeness in the *TP*.

[24] For other instances of AA magic wands, see O'Donnell (2016a p.XX).

[25] See also O'Donnell (2014; 2016a; 2016b). No such inconsistency exists in orthodox theory which relies on perfect knowledge assumptions.

[26] Davidson's (2002 pp. 41-2) claim that 'all mainstream economists believe that in the long run, money is neutral', [a view that] 'classical theorists have always believed is a universal truth' is thus inconsistent with his own citations of mainstream views.

[27] Often in Davidson's writings a completely incompatible fast-learning definition is substituted for the scientific definition. See O'Donnell (2014; 2016b).

[28] The same argument applies to ergodic systems, so that the same internal inconsistency exists within AA's account of orthodoxy.